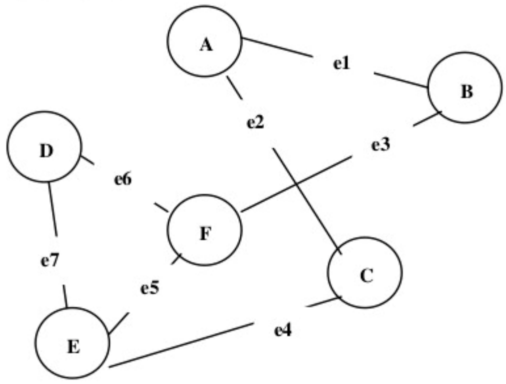
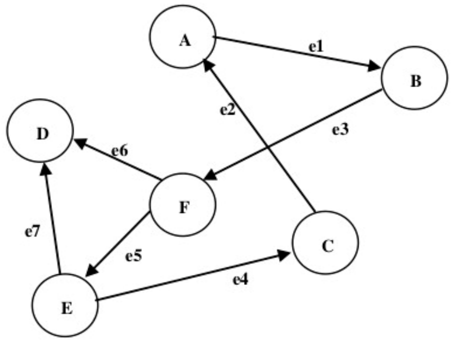
Exercises Series N°: 1 and 2

Exercise 1: Consider the following graph:



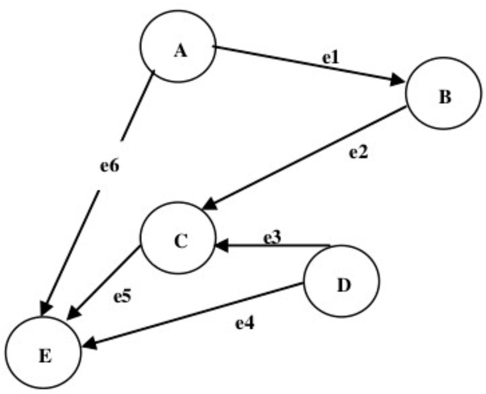
1. Give the degree of each vertex and the degree of the graph.
2. Is the graph connected? If not, determine its connected components.
3. Give the subgraph induced by the set of vertices A, C, D, F.
4. Give the partial subgraph induced by the set of edges e1, e3, e4, e5.
5. Provide a chain belonging to the graph.
6. Provide a cycle belonging to the graph.

Exercise 2: Consider the following directed graph:



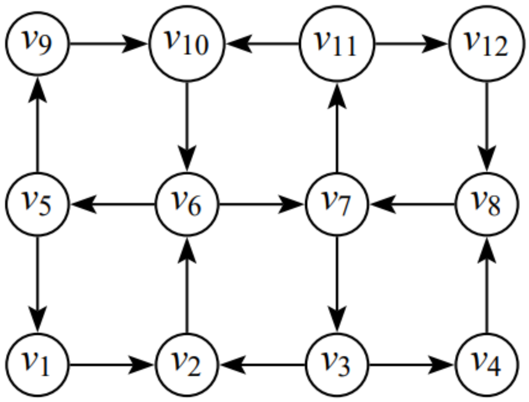
1. Determine the in-degree and out-degree of each vertex.
2. Is there a path from vertex E to vertex B?
3. What is the distance between vertex A and vertex D?
4. What is the distance between vertex D and vertex A?
5. Is the graph strongly connected? If not, determine its strongly connected components.

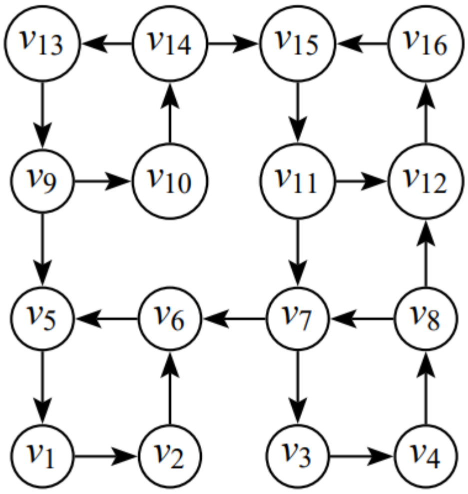
Exercise 3: Given the following graph, determine the incidence matrix, the adjacency matrix, and the adjacency list of the graph.



Exercise 4:

1. Are the graphs below strongly connected? If not, provide their strongly connected components.
2. Propose an algorithm to determine if a graph is strongly connected.





Exercise 5: A computer network is composed of 4 machines. Each machine can send data to other machines with a maximum bandwidth specified in the following table:

Machine	**Sends to (machine, bandwidth)**
1	(4, 100)
2	(3, 128), (4, 1000)
3	(1, 60)
4	(2, 500)

1. Represent this network using a graph.
2. What data structure can be used to store it in memory?

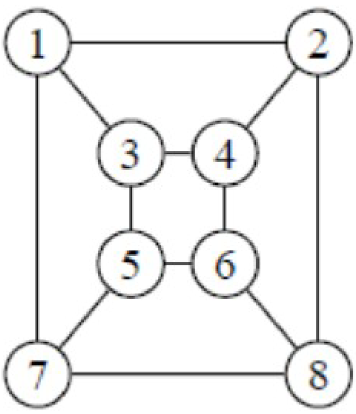
Exercise 6: Prove that the sum of the degrees of the vertices in a simple graph is equal to twice the number of edges.

Exercise 7 Show that a simple graph has an even number of vertices with odd degrees.

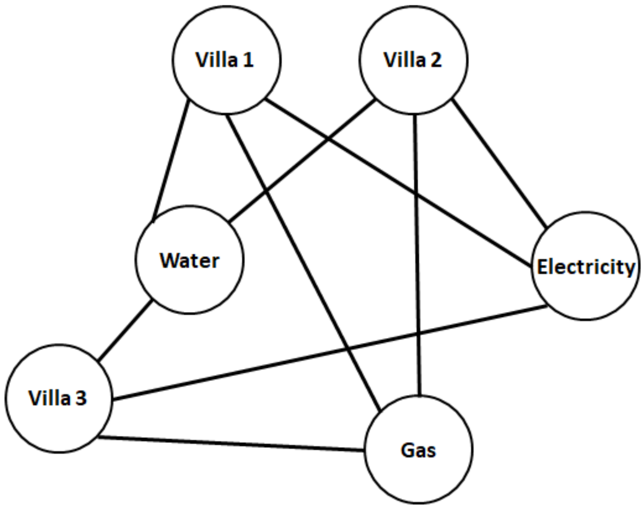
Exercise 8 Is it possible to connect 15 computers such that each device is connected to exactly three others?

Exercise 9 Prove that a graph is bipartite if and only if it contains no odd-length cycles.

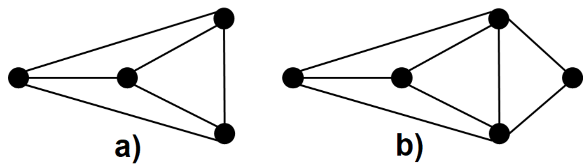
Exercise 10 Is this graph bipartite?



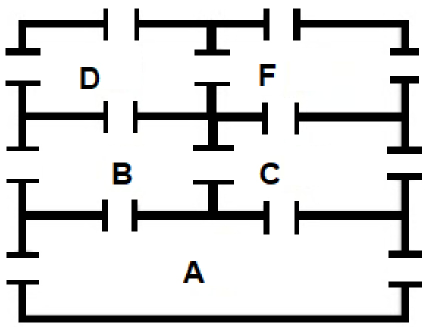
Exercise 11 Using the property demonstrated by Manori, show that the graph below, corresponding to the connections of villas to water, gas, and electricity, is not planar.



Exercise 12 A postman wants to complete his route without traversing any street twice. Is this possible if his route is described by the following two graphs (where each street is represented by an edge)?



Exercise 13 A security agent is in room D and must visit all rooms, passing through each door exactly once.



Exercise 14 Six students (A, B, C, D, E, F) need to take exams, each exam taking half a day:

- Algorithmics: students A and B
- Compilation: students C and D
- Databases: students C, E, F, and G
- AI: students A, E, F, and H
- Architecture: students B, F, G, and H

We want to organize the exam session in the shortest possible time. Model this problem using a graph and reformulate it within that framework.

Exercise 15 Three professors (P1, P2, P3) need to give lectures next Monday to three classes (C1, C2, C3):

- P1 needs to give 2 hours of lectures to C1 and 1 hour to C2.
- P2 needs to give 1 hour to C1, 1 hour to C2, and 1 hour to C3.
- P3 needs to give 1 hour to C1, 1 hour to C2, and 2 hours to C3.

1. How can this situation be represented using a graph? What type of graph do you obtain?
2. How many time slots are needed at a minimum?
3. Use the graph to propose a schedule for Monday for these professors.