

Graph theory

Scheduling problems

Introduction to scheduling problems

- **Scheduling problems** are a category of optimization problems in which tasks must be assigned to resources over time, subject to various constraints.
- Scheduling a set of tasks means planning their execution by allocating resources and setting start dates.
- Scheduling problems appear in all areas of the economy:
 - computing (tasks are programs; resources are processors, memory, etc.),
 - construction (project monitoring),
 - industry (workshop activities in production management and logistics issues),
 - administration (timetables).

Introduction to scheduling problems

- **Key Concepts in Scheduling**

- **Resources:** Technical or human means with limited or unlimited availability known a priori.

- **Tasks:** Elementary work requiring a specific duration and number of resource units.

- **Objectives of Scheduling:** Solving a scheduling problem must reconcile two objectives:

- **Static aspect:** Create a work plan based on forecast data.

- **Dynamic aspect:** Make real-time decisions based on resource status and task progress.

- **Questions Addressed by Scheduling:**

- Best task and resource organization to meet deadlines.
- Minimum time needed to complete the project.
- Earliest and latest start and finish times for each task.
- Execution order of project activities and activation schedule respecting constraints.

- **Prominent Scheduling Methodologies**

- **Gantt charts** :used for clear visualization of schedules and tracking progress.
- **PERT method** :used for deeper analysis of dependencies and time-critical tasks, especially in complex projects. Both can be combined for comprehensive project management.

GANTT chart

- **Definition of Gantt Chart:**

- A Gantt chart is a visual scheduling tool that displays project tasks along a timeline, illustrating their start and finish dates.
- It consists of a vertical axis listing tasks and a horizontal axis representing time intervals, with bars indicating the duration of each task

- **Importance of Gantt Charts**

- Facilitates comprehension of project structure.
- Enables straightforward identification of roles and responsibilities for each participant.
- Helps visualize task sequences and simultaneous operations, enhancing project planning and tracking

GANTT chart : Early and Late Scheduling

- Two primary methods of scheduling are Early Scheduling and Late Scheduling, each with distinct approaches to task organization.
- **Early Scheduling** : Early scheduling involves initiating tasks as soon as possible, focusing on those without any preceding tasks.
- **Late Scheduling** : Late scheduling focuses on positioning tasks as late as possible, particularly those without successors.

	Early Scheduling	Late Scheduling
Start Timing	As early as possible	As late as possible
Focus	Tasks without predecessors	Tasks without successors
Representation Order	Sequentially from start	Sequentially from end

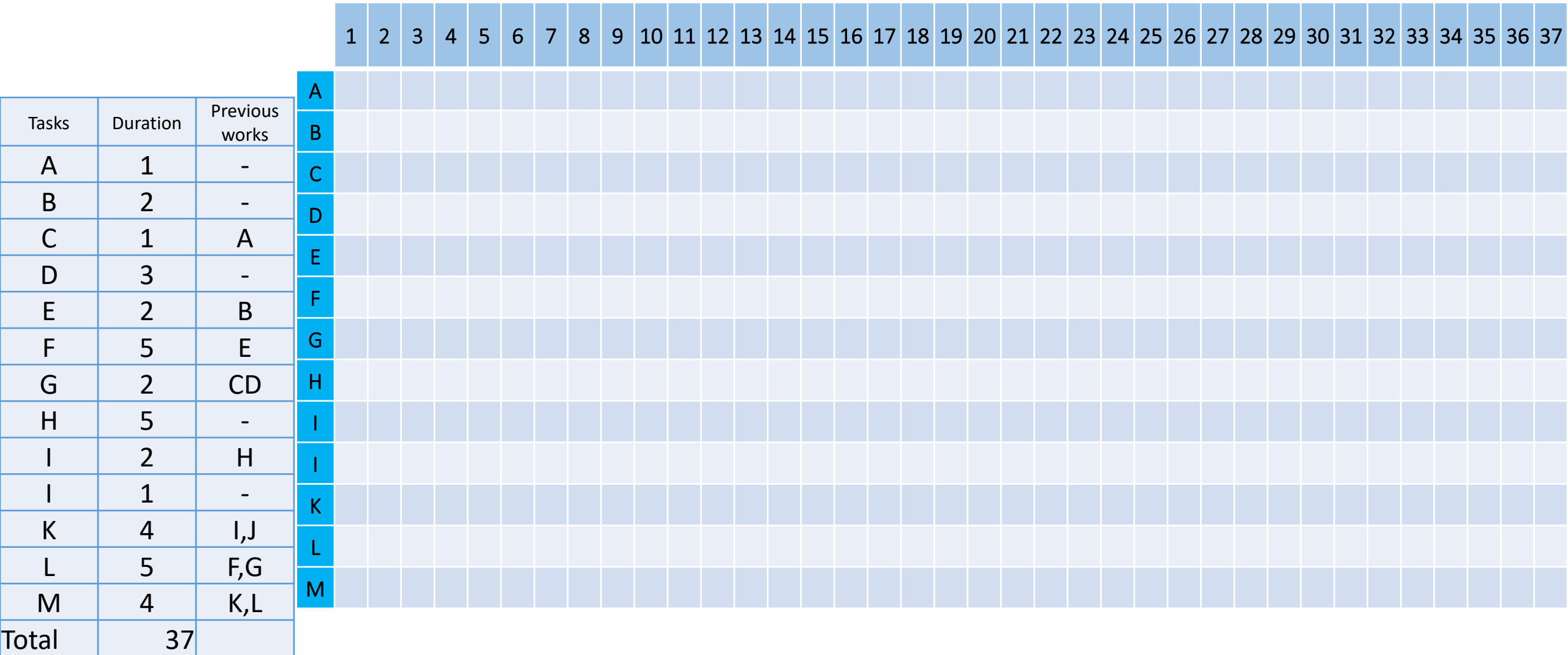
Steps to Create a Gantt Chart

- 1. Determine Tasks and Their Durations** : List all tasks required for the project and estimate the time needed for each.
- 2. Identify Task Sequences and Concurrent Tasks** : Establish the order of tasks and identify which tasks can be performed simultaneously.
- 3. Graphical Representation** : Graphical representation of the table using software such as word processing, spreadsheet or project management software (such as Microsoft Project)
 - Insert a table to represent tasks and timelines.
 - Use horizontal bars to represent task durations visually.
 - Adjust bars to reflect start dates, end dates, and dependencies

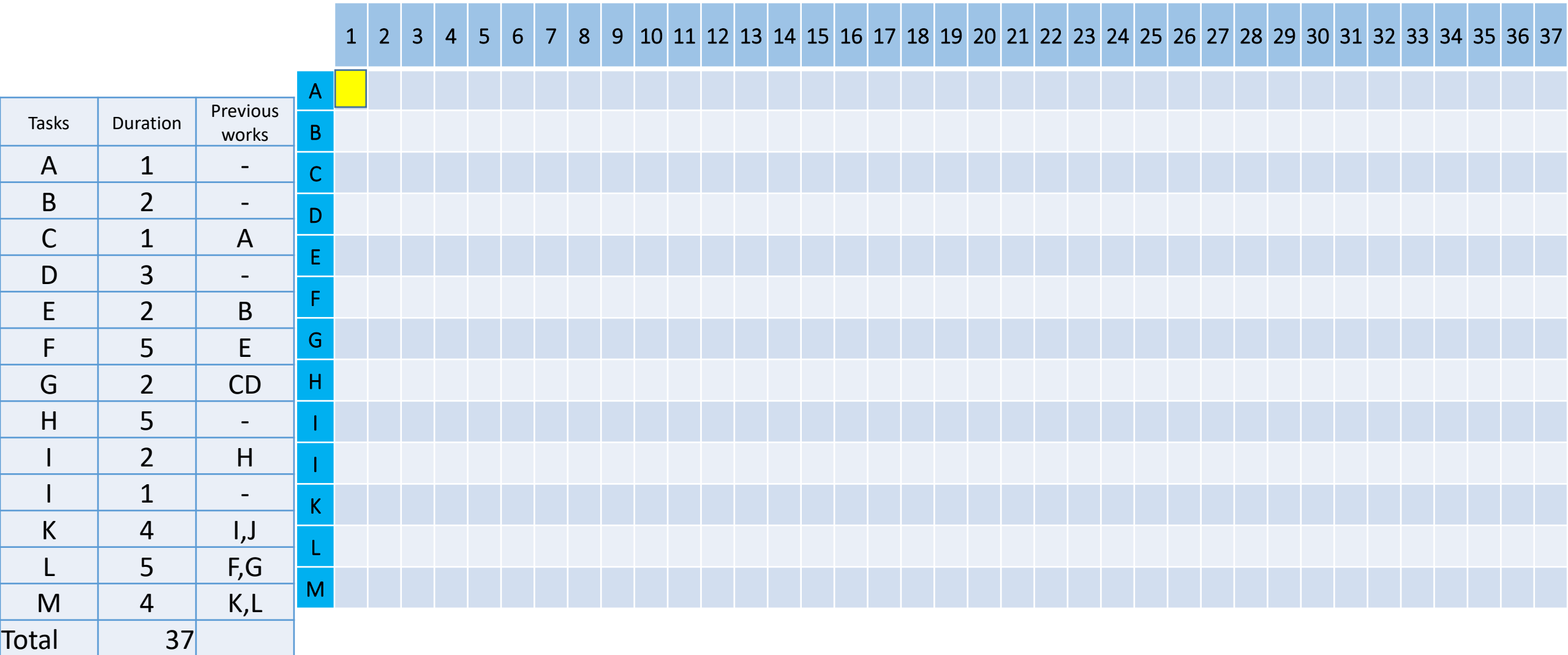
Gantt chart: "Early Scheduling"

Tasks	Duration	Previous works
A	1	-
B	2	-
C	1	A
D	3	-
E	2	B
F	5	E
G	2	CD
H	5	-
I	2	H
I	1	-
K	4	I,J
L	5	F,G
M	4	K,L
Total	37	

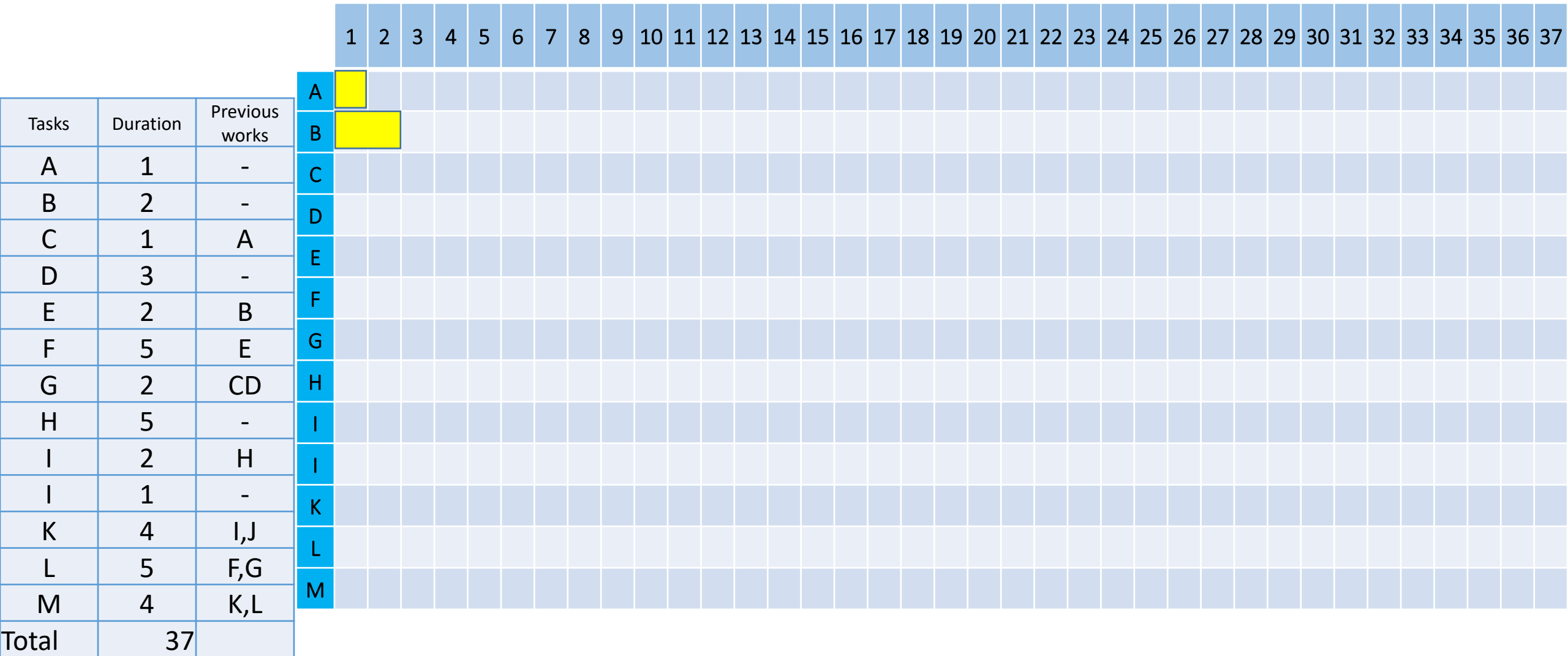
Gantt chart: “*Early Scheduling*”



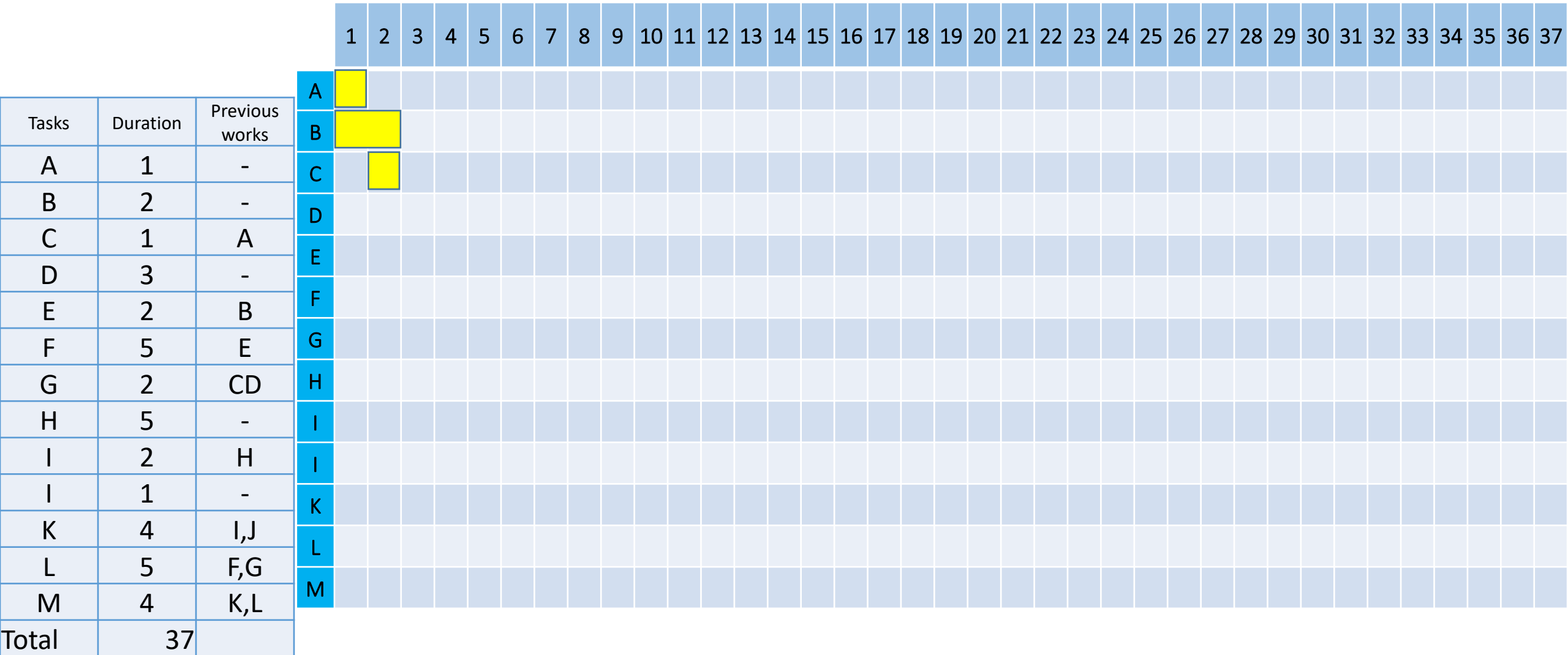
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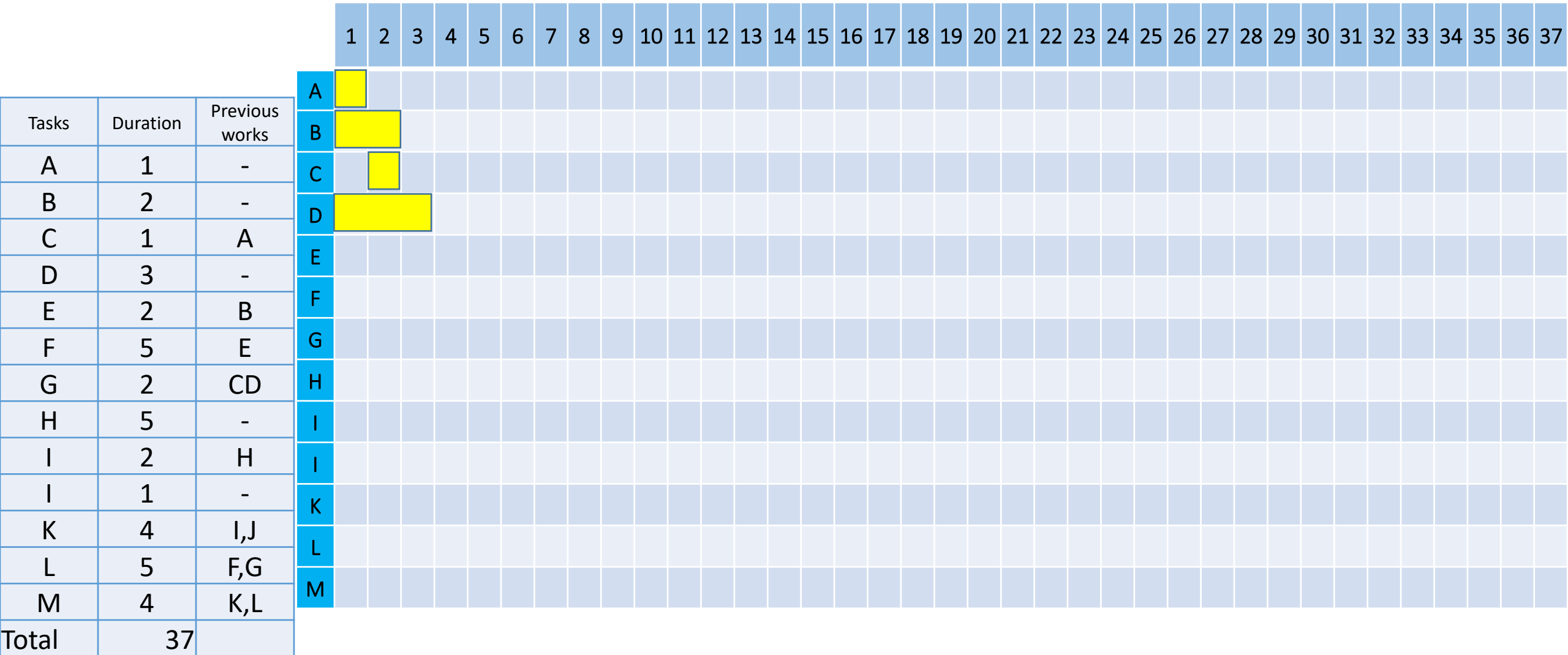
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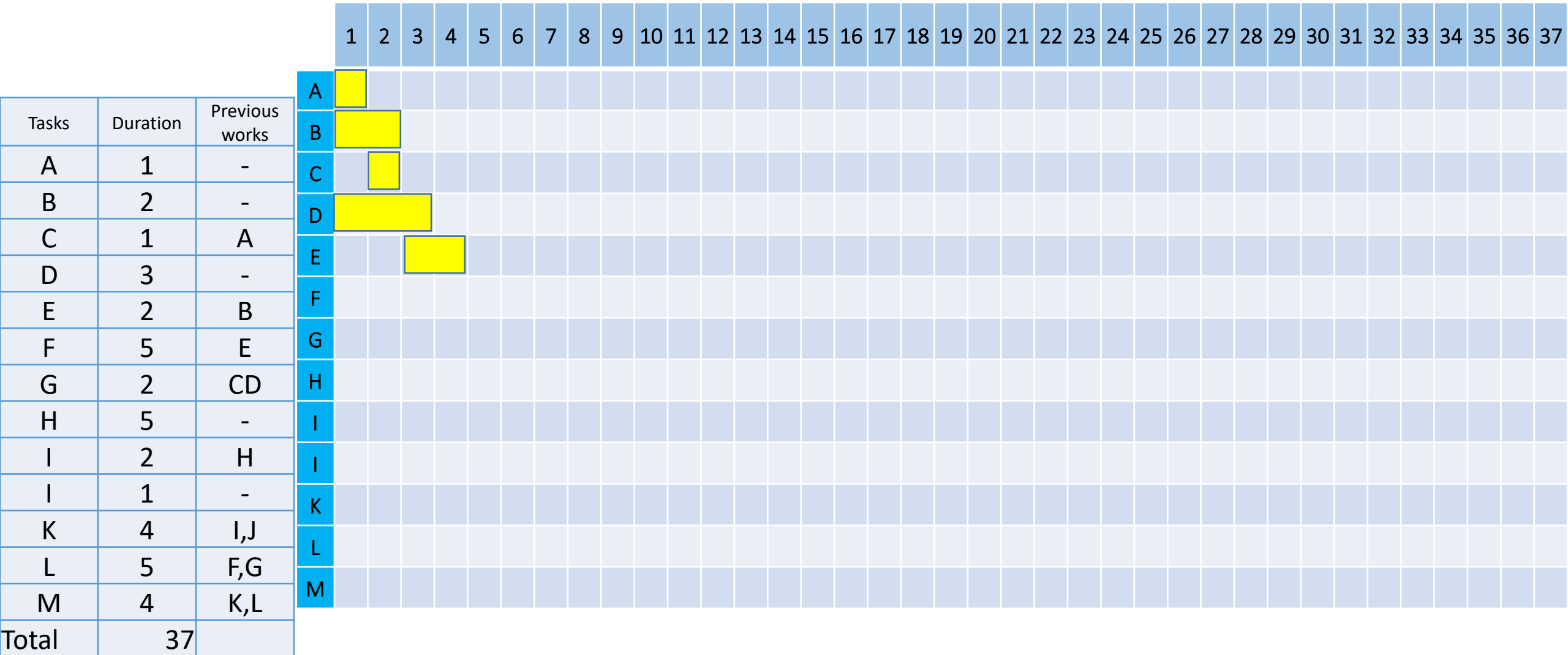
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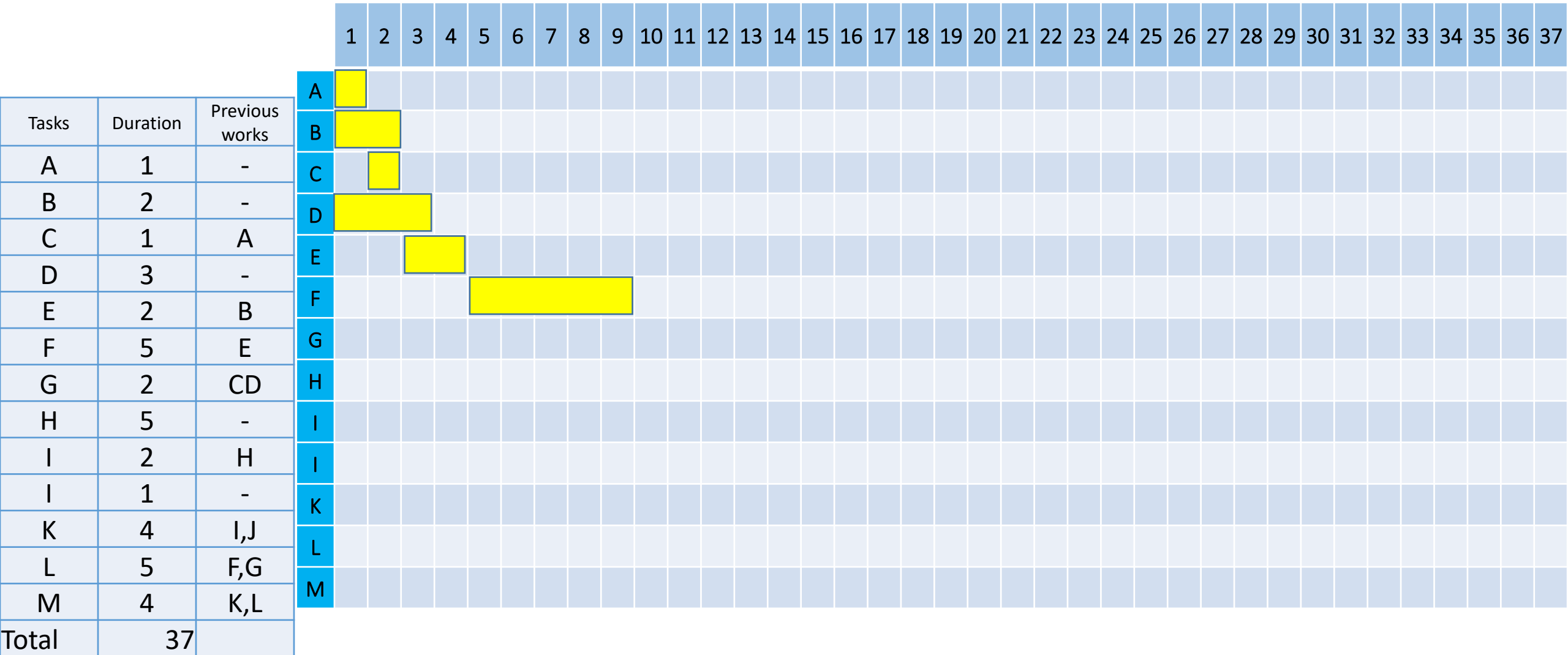
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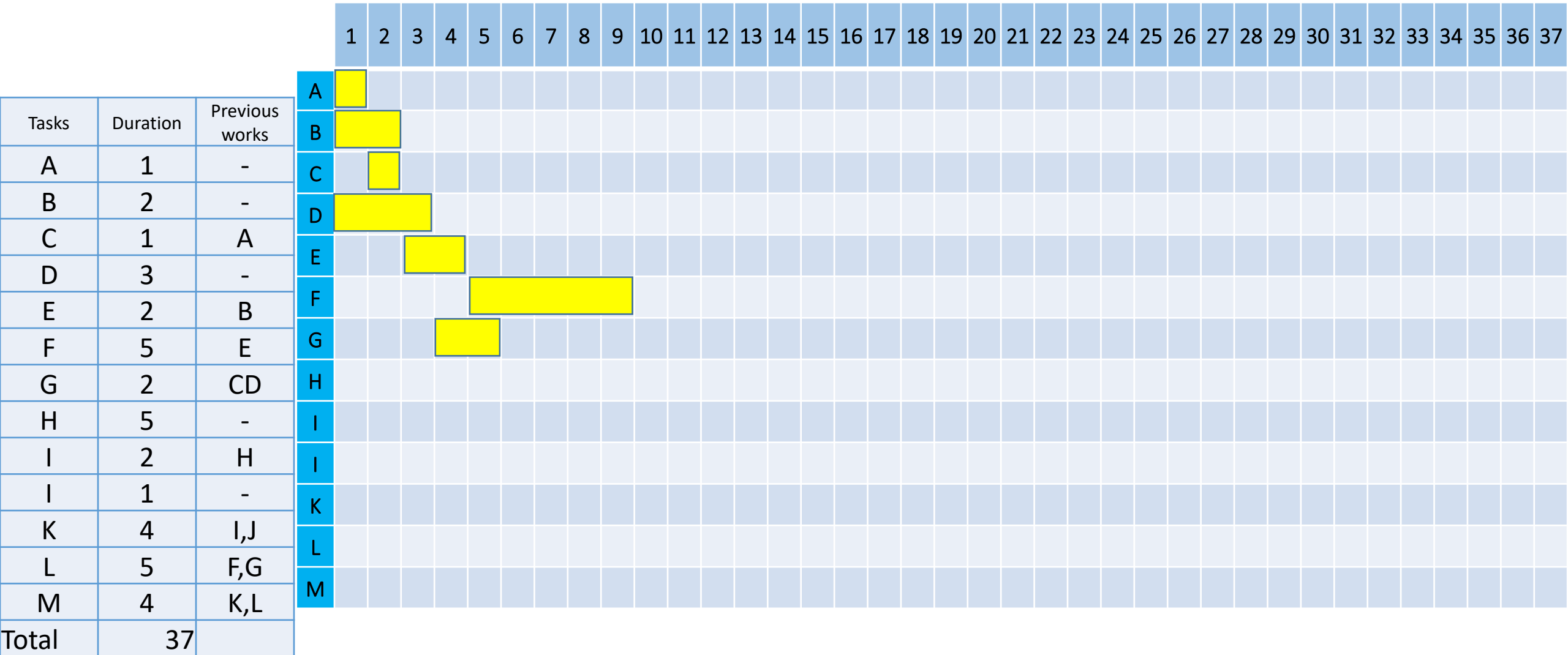
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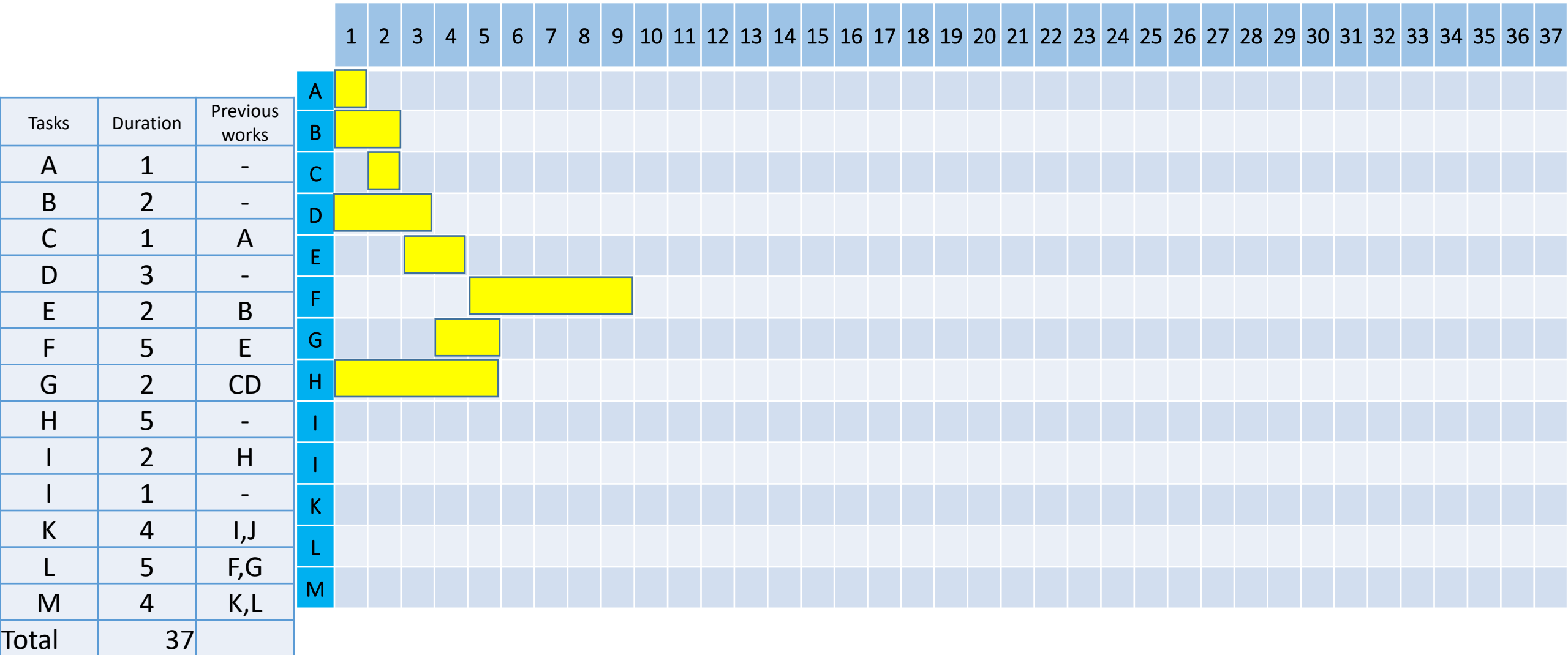
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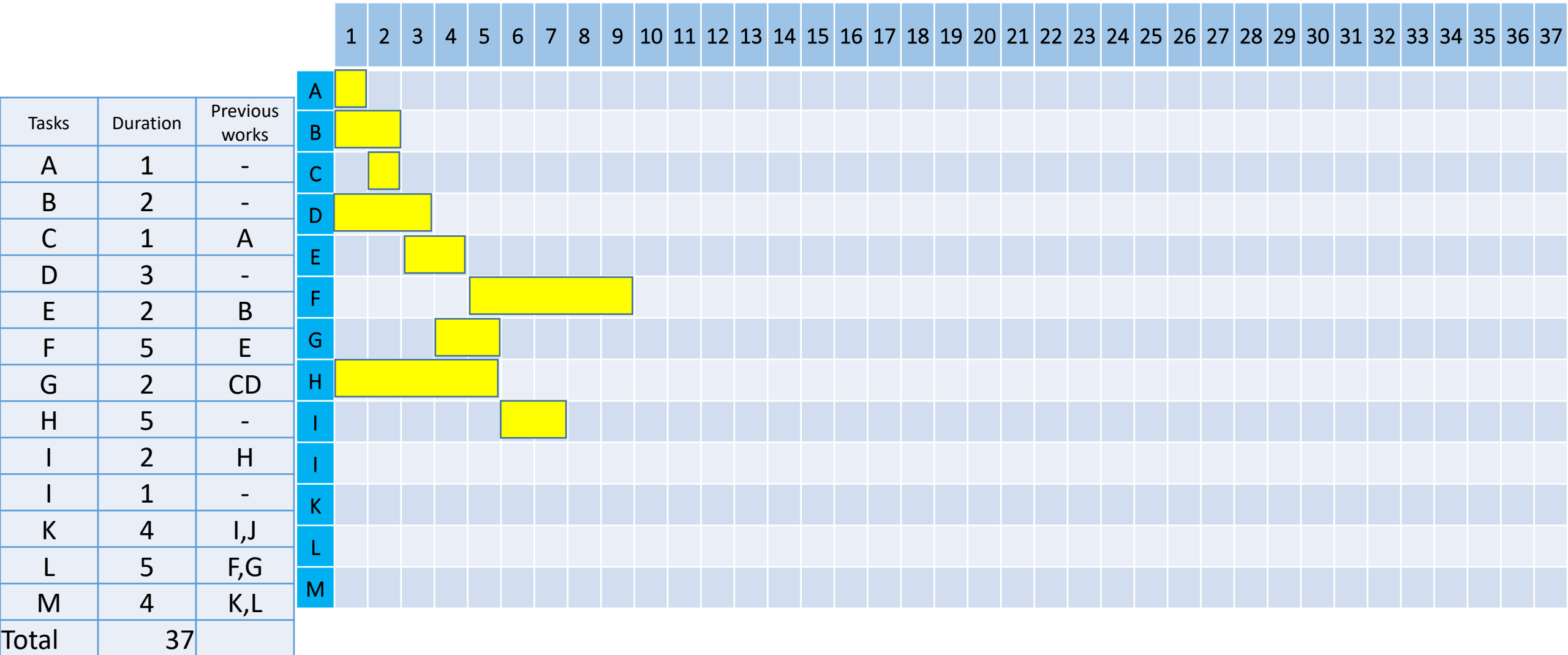
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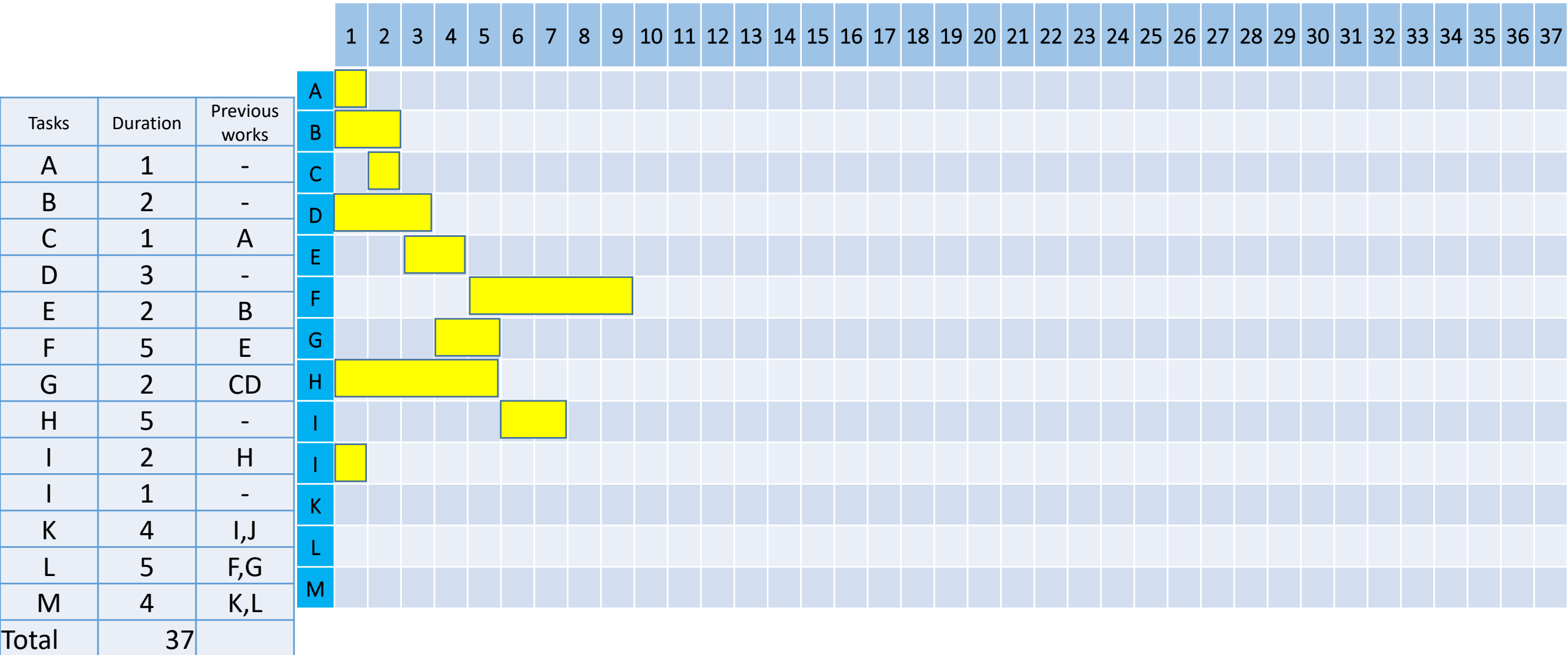
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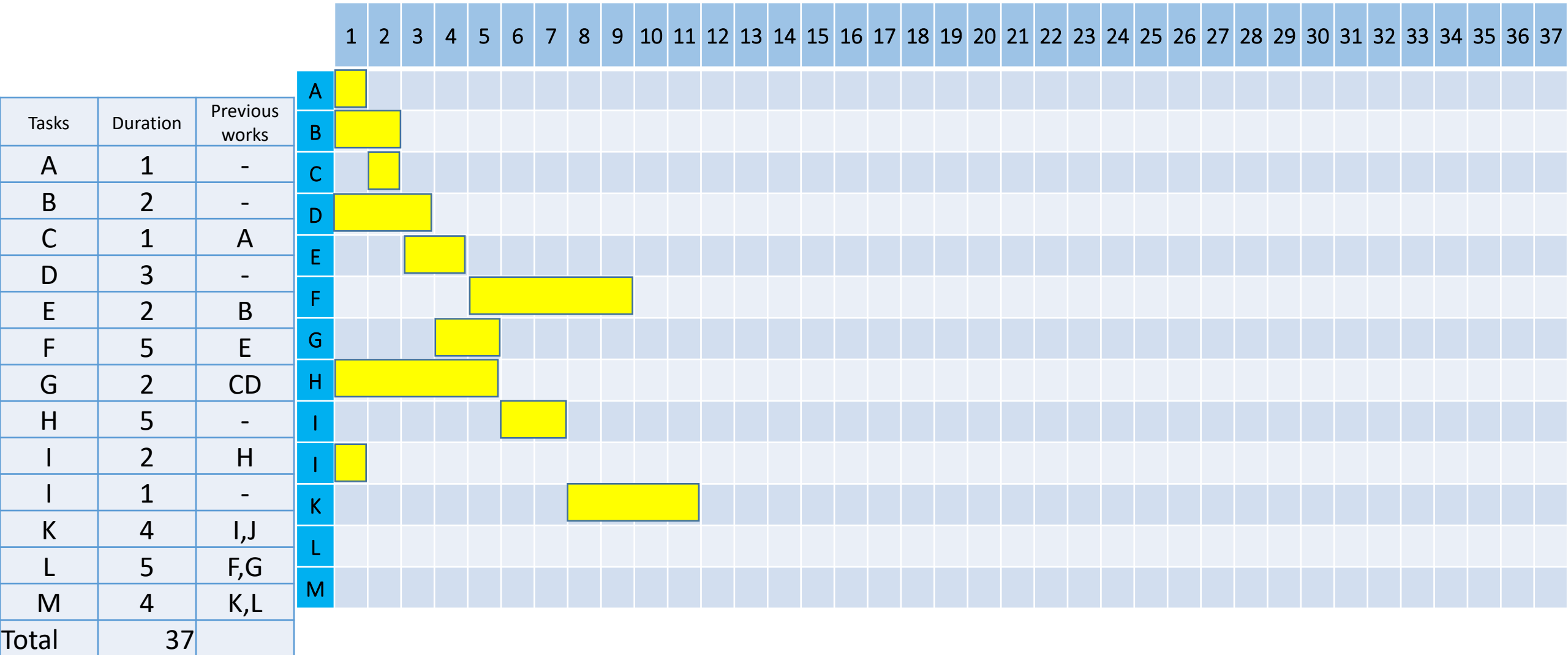
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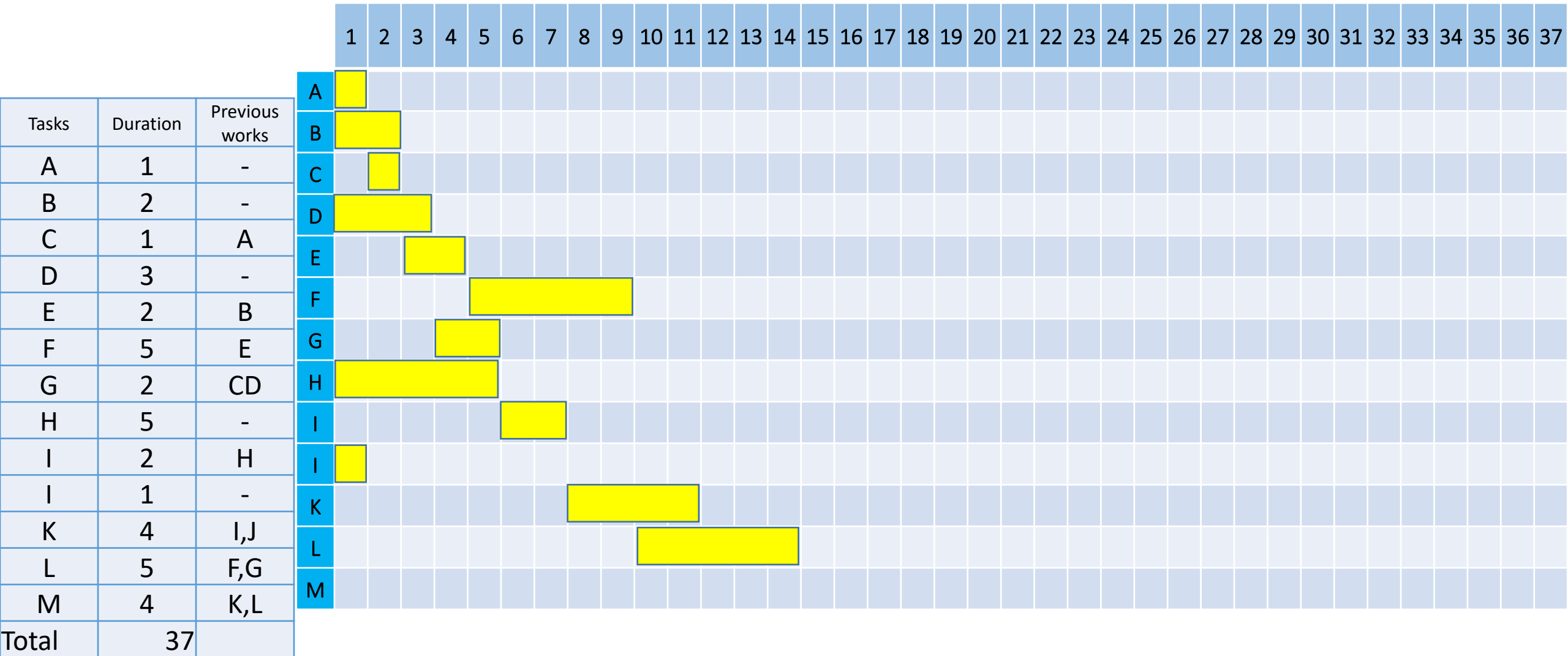
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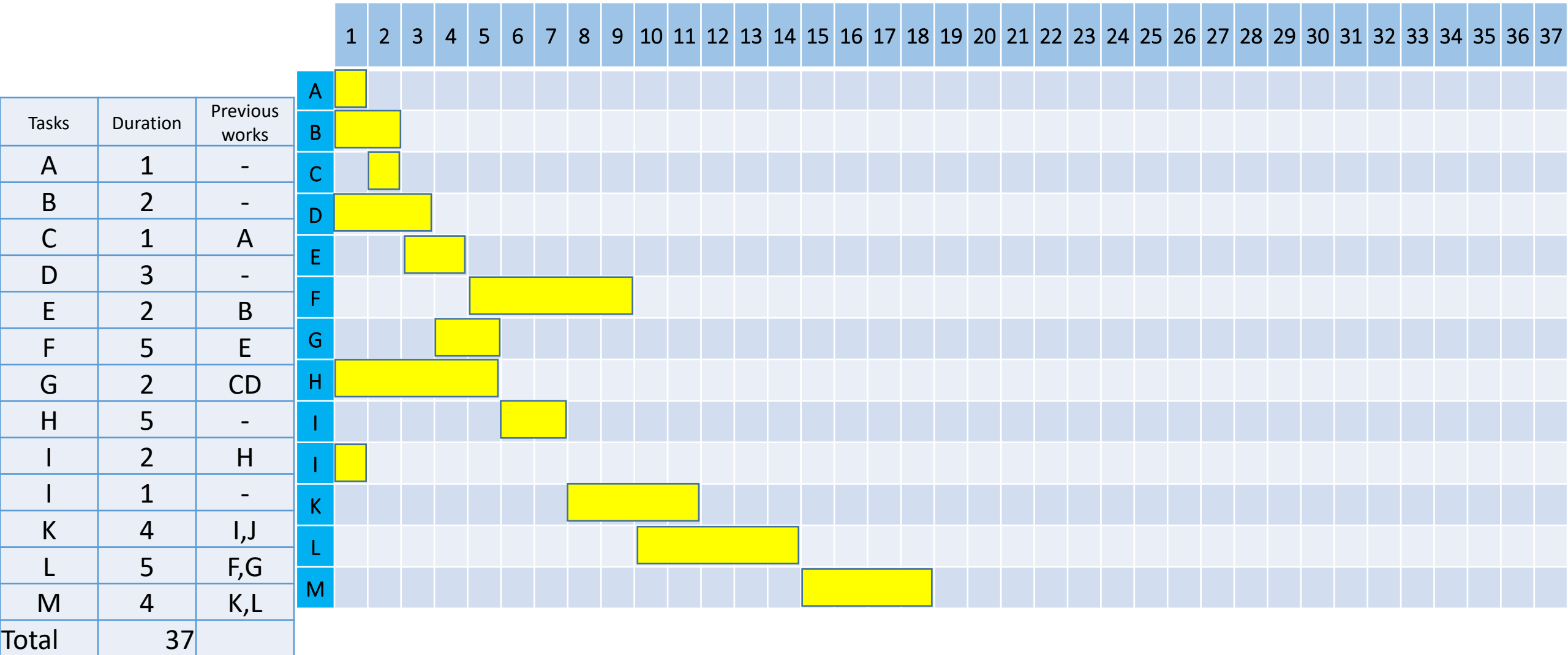
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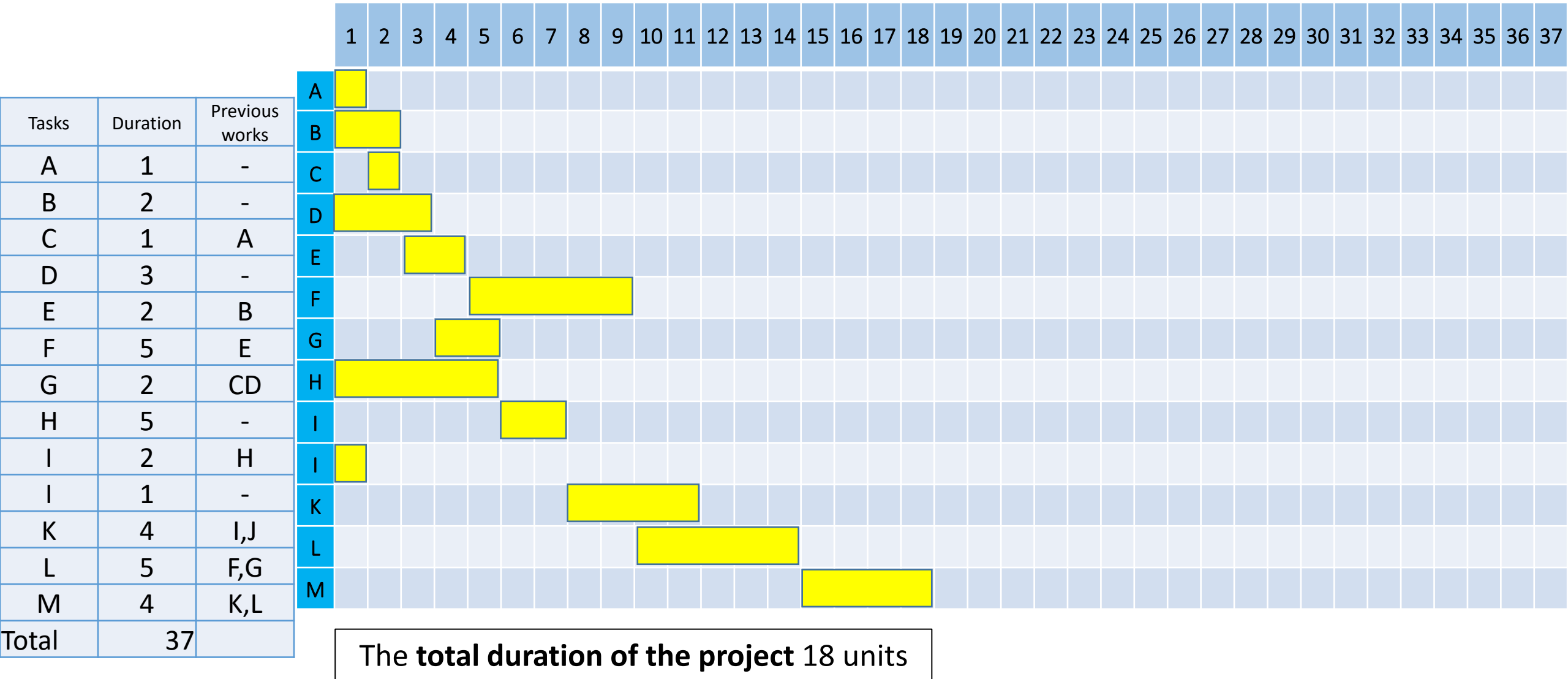
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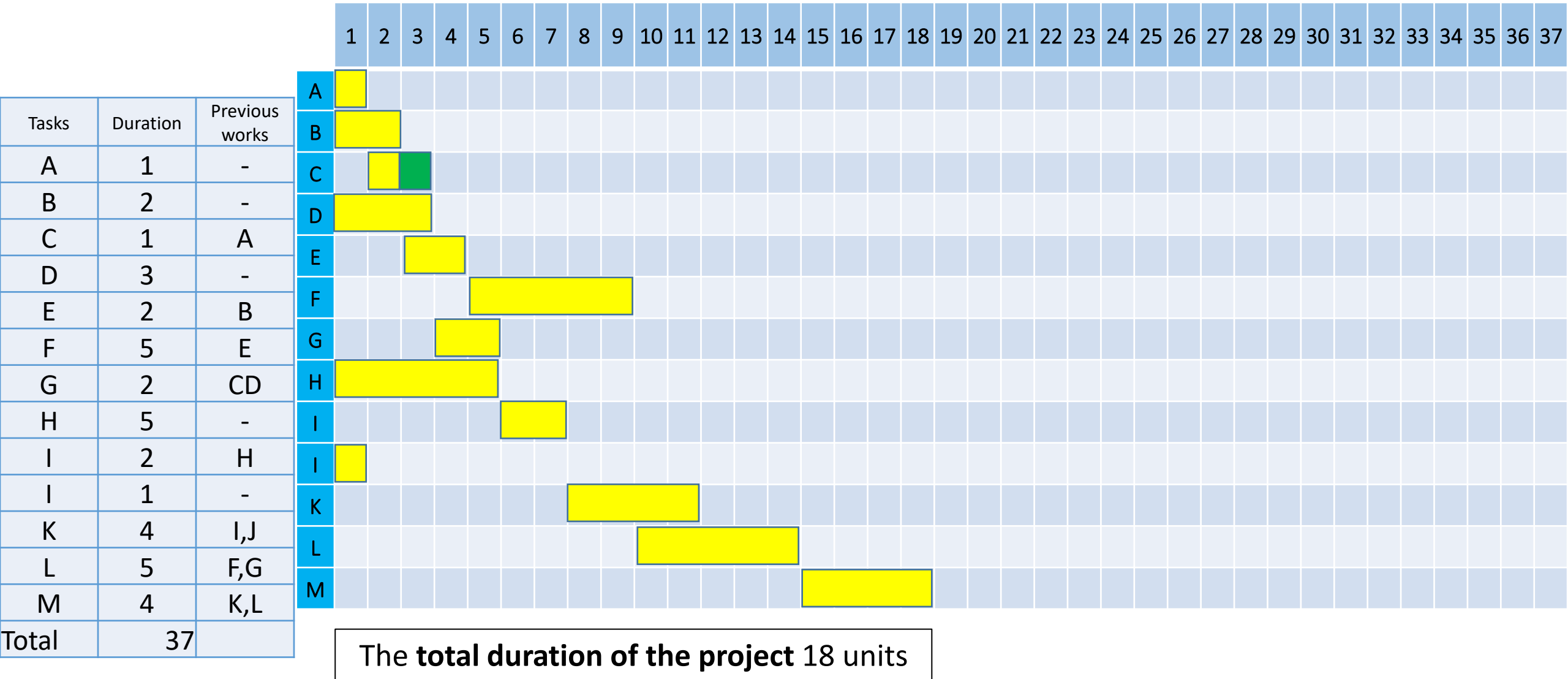


Gantt chart: “*Early Scheduling*”



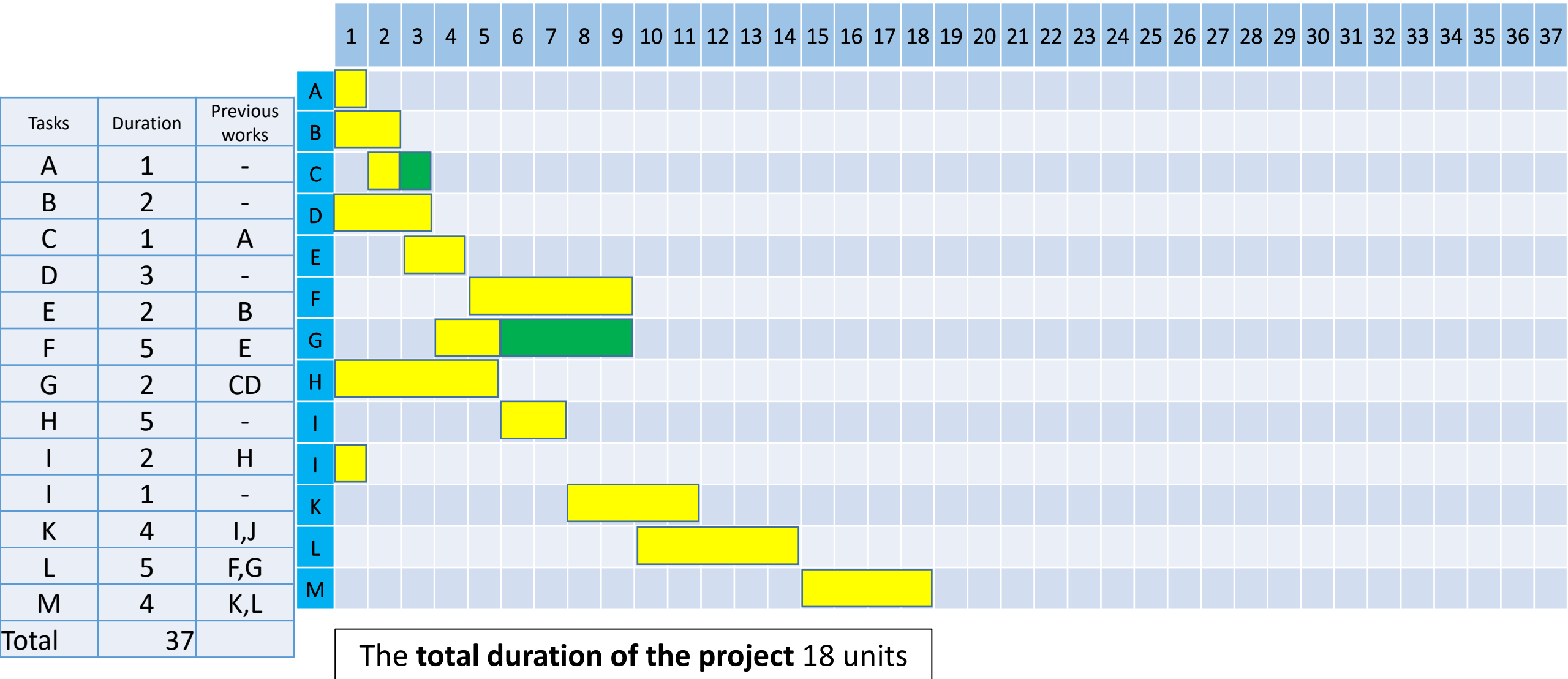
The **total duration of the project** 18 units

Gantt chart: “*Early Scheduling*”

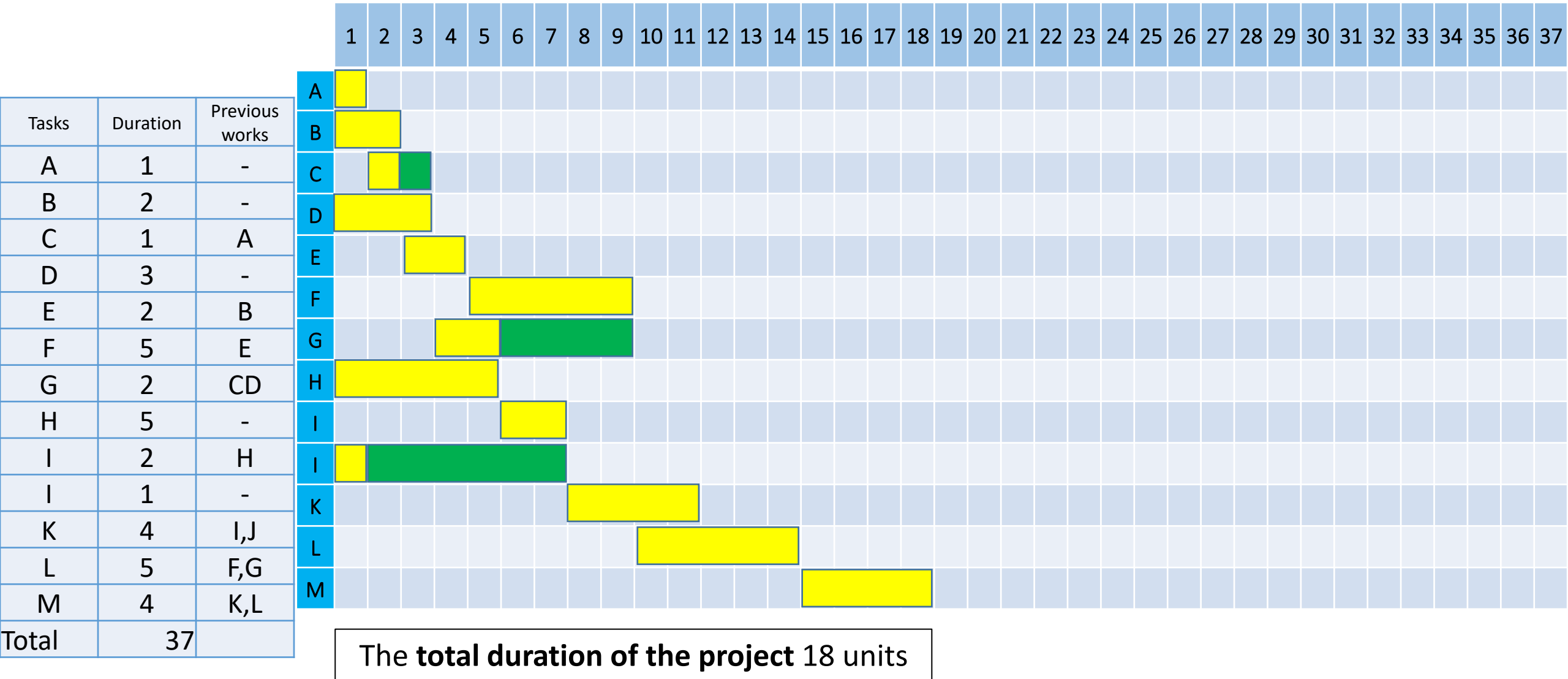


The total duration of the project 18 units

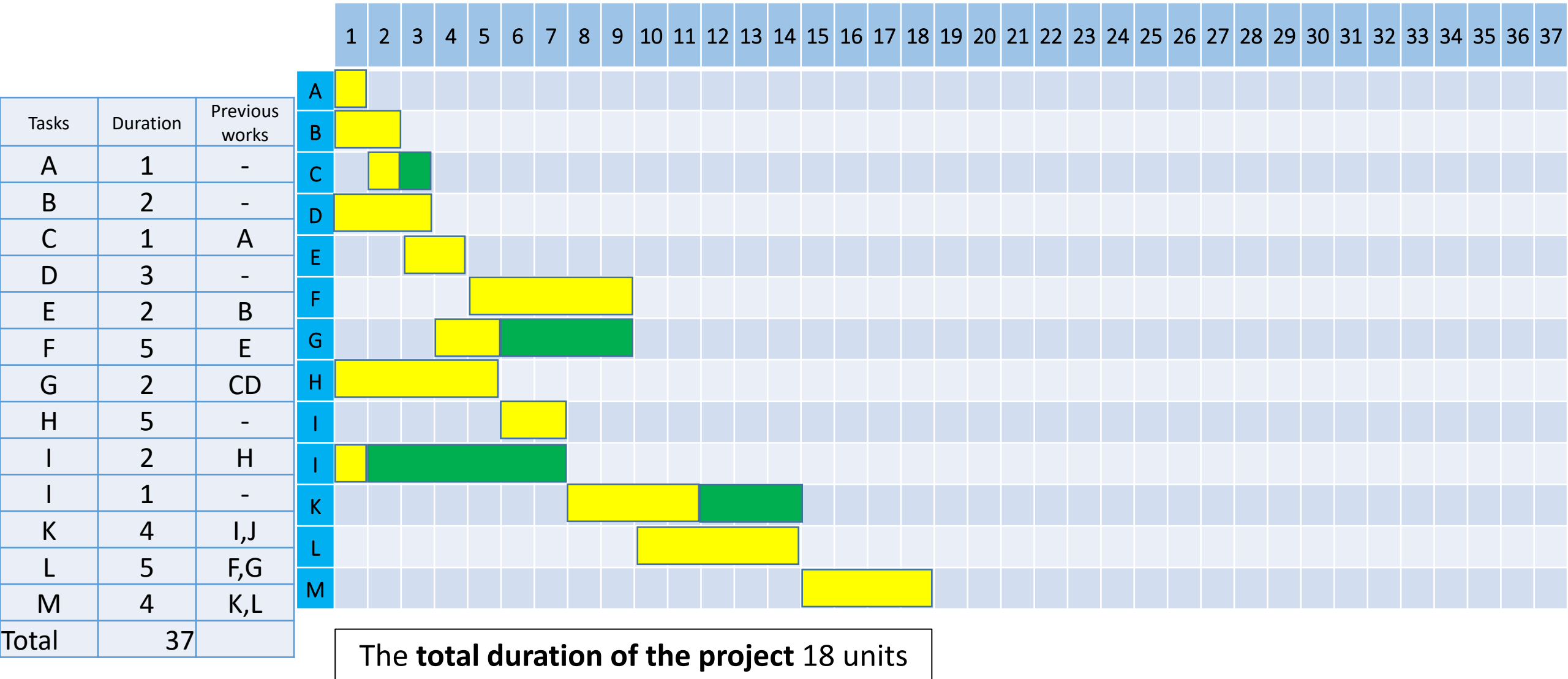
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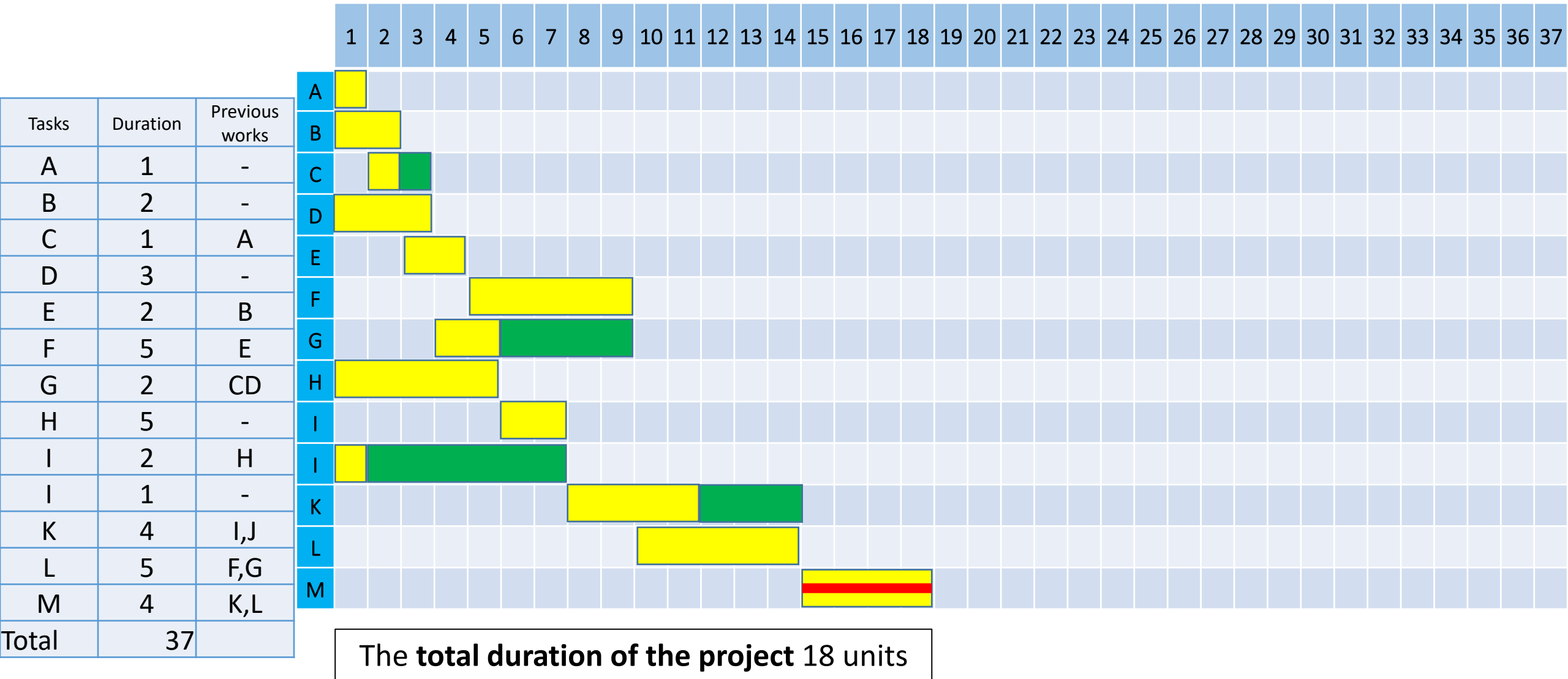
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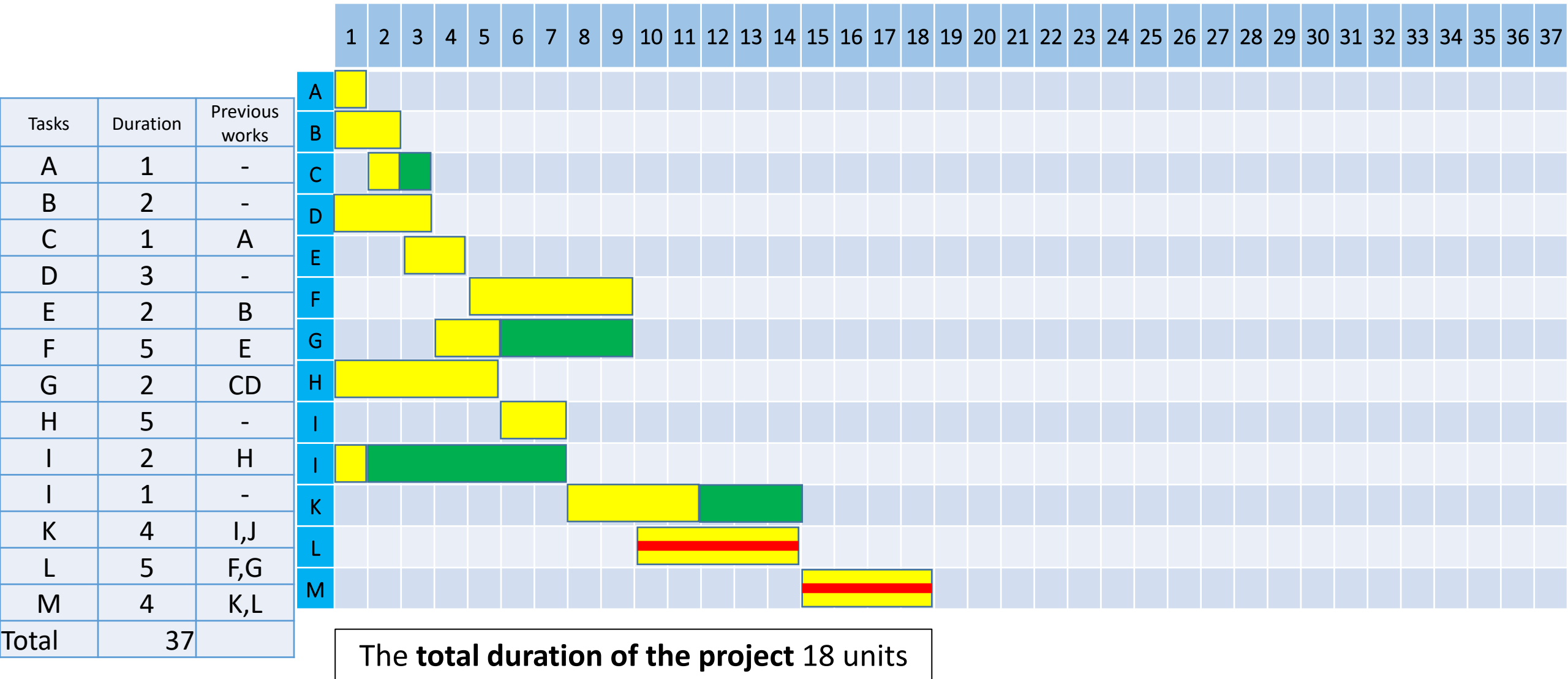
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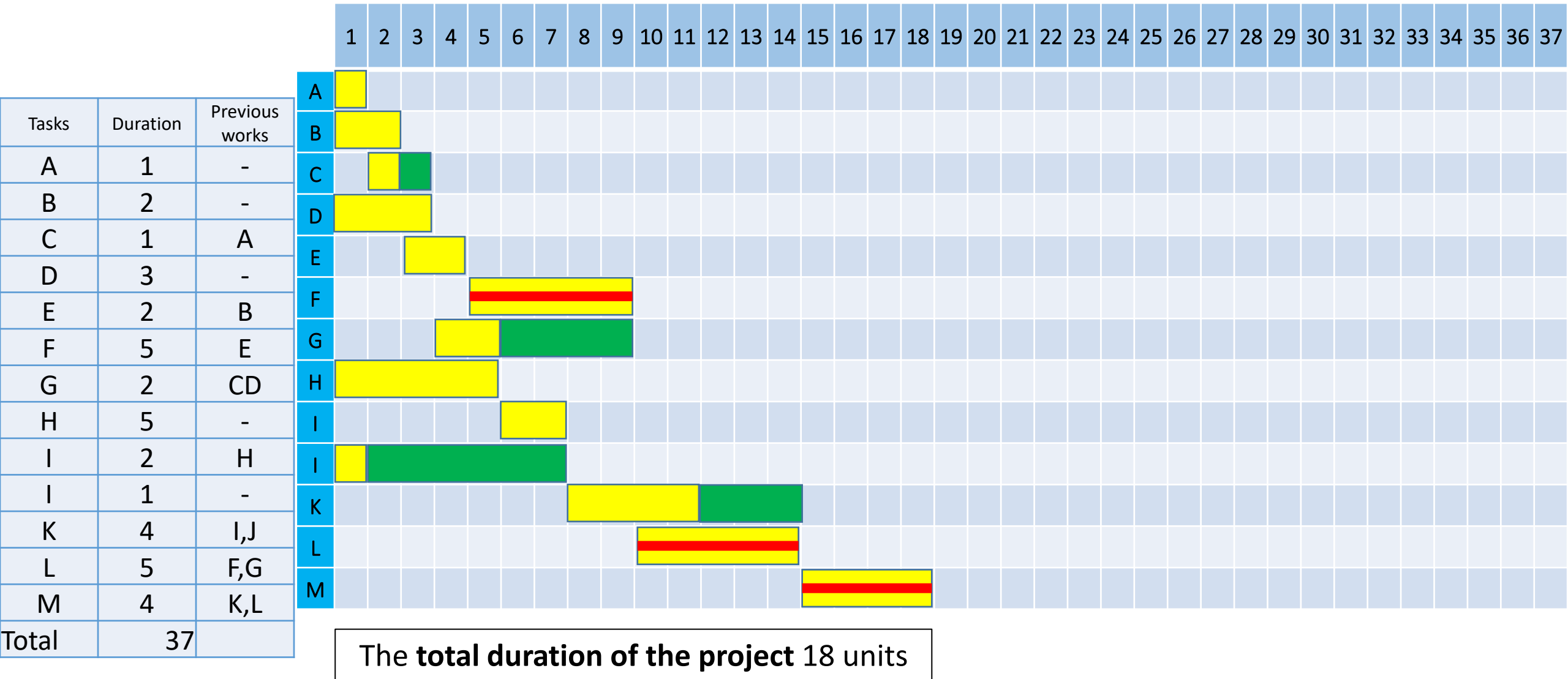
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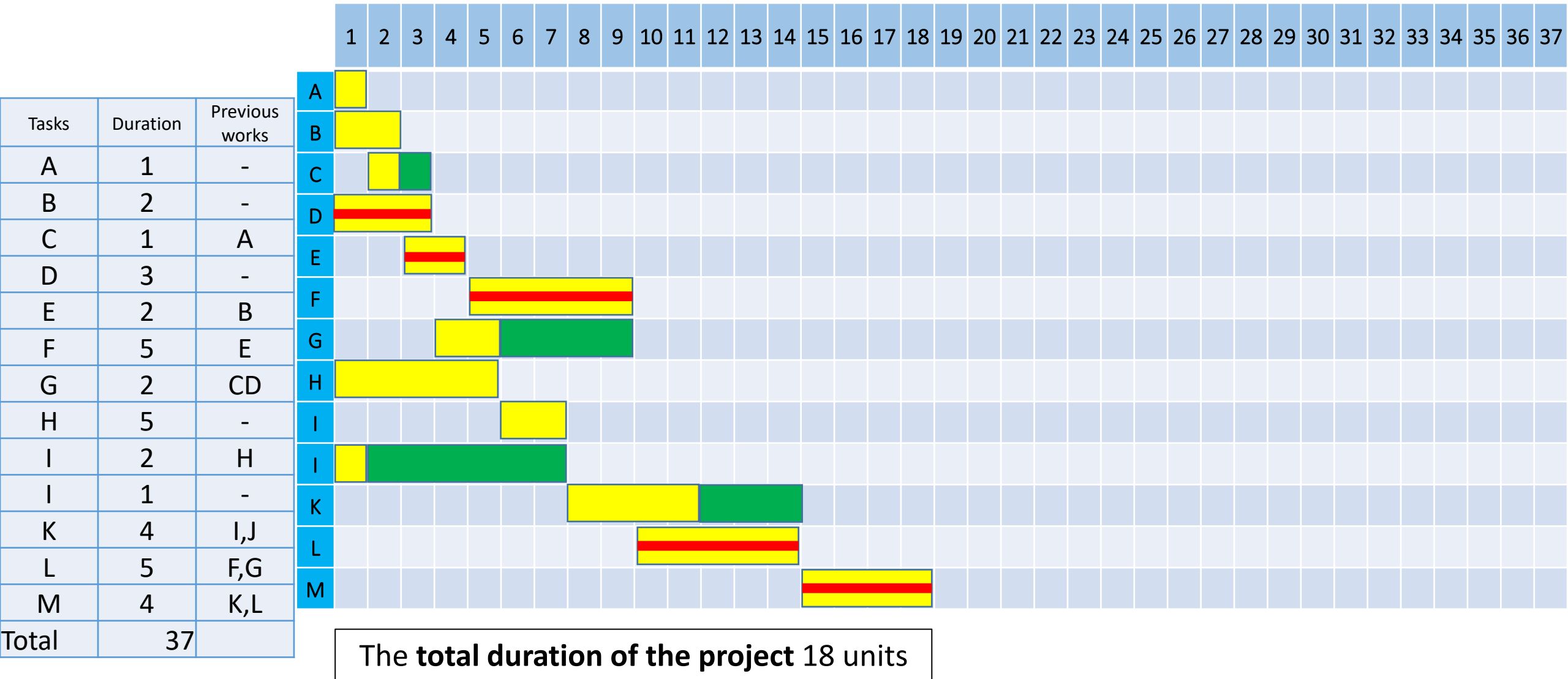
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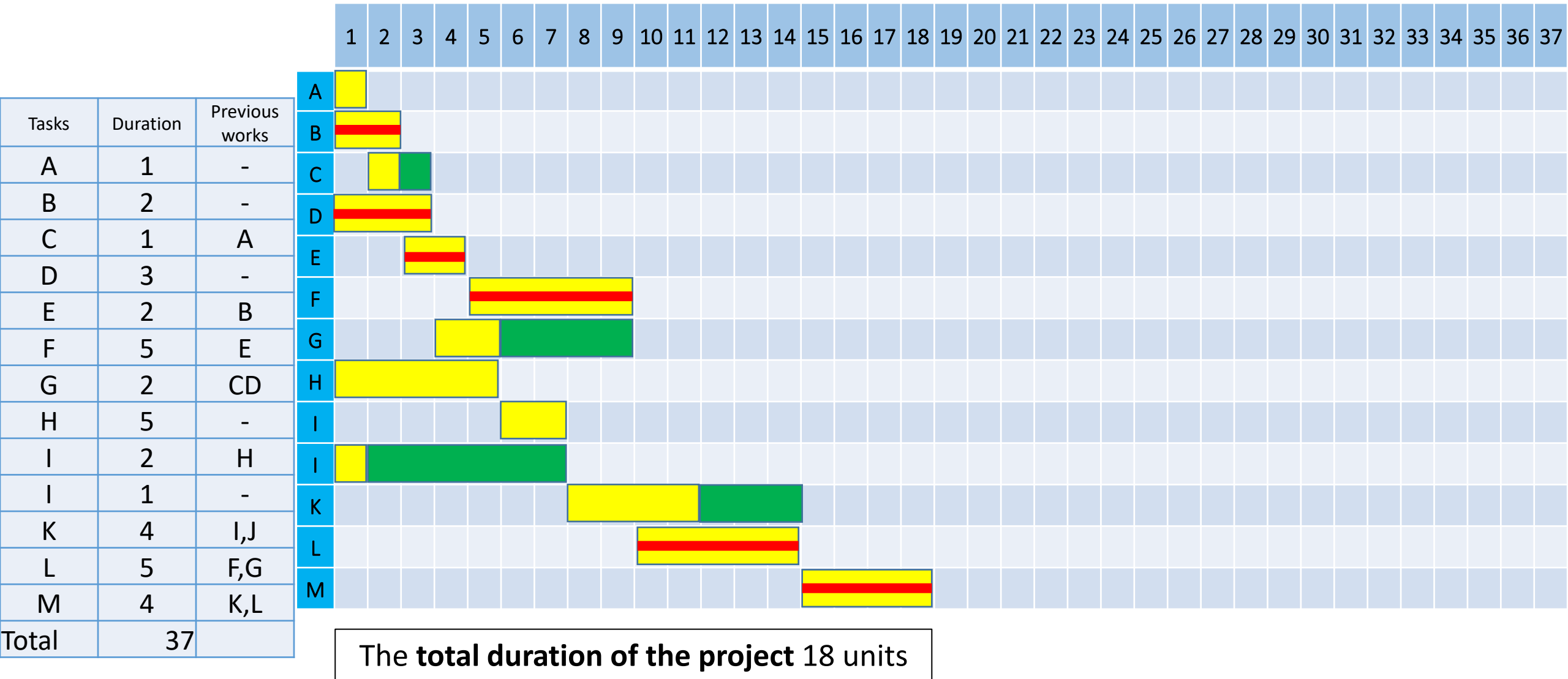
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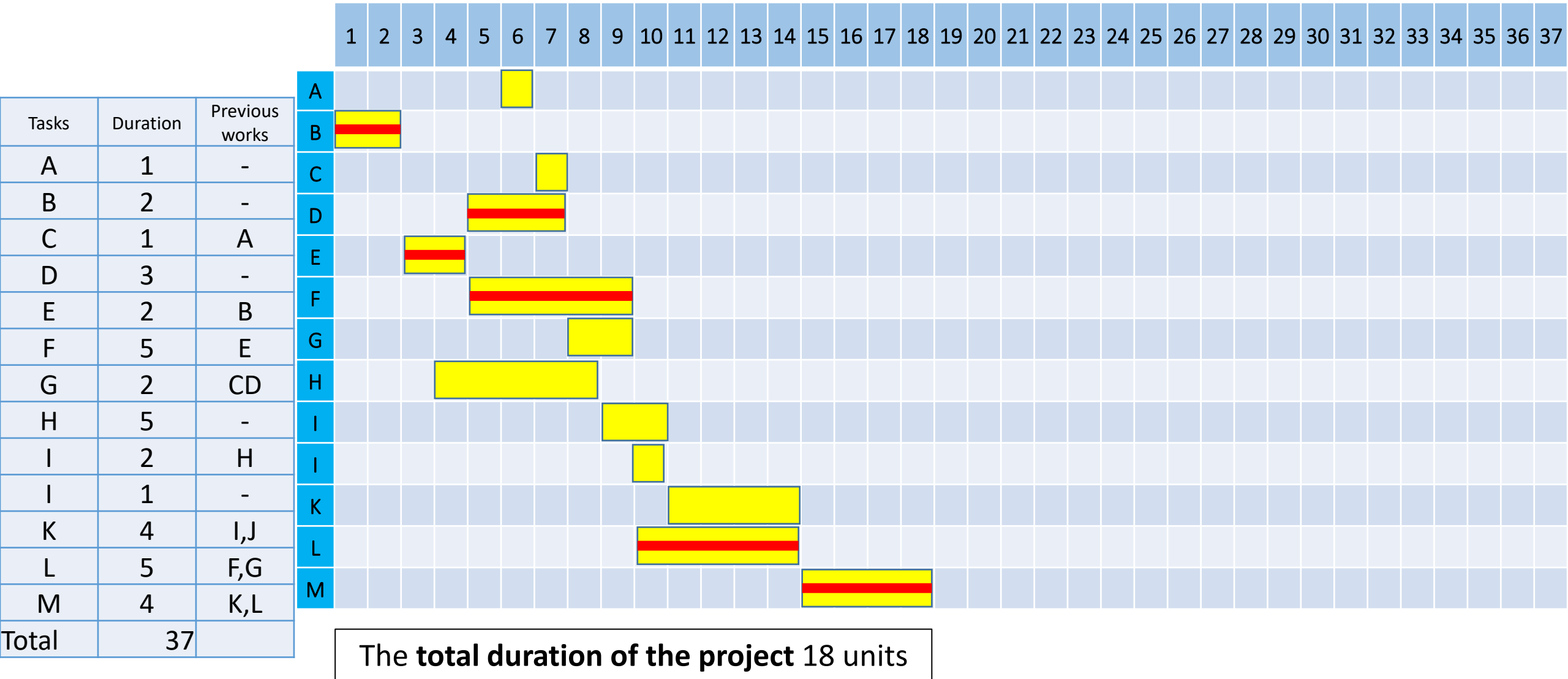
Gantt chart: “*Early Scheduling*”



Gantt chart: “*Early Scheduling*”



Gantt chart: "Late Scheduling"



PERT planning method

- **P.E.R.T** : **P**rogram **E**valuation **R**eview **T**echnique
- This method was developed in the United States in 1957 during the development of the POLARIS missile.
- This project involved 250 prime contractors and about 9,000 subcontractors.
- The originally planned duration of 6 years was reduced to 2 and a half years.

PERT(Approach)

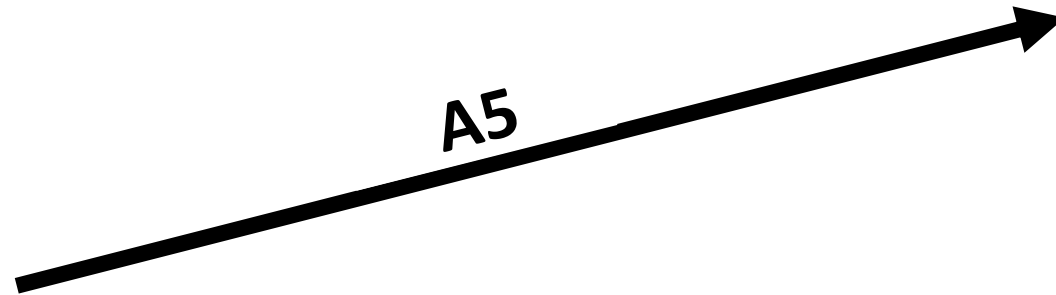
The PERT method is a method of organizing tasks in a network according to their dependency and chronology, with the objective of obtaining a finished product.

- **Step 1** : Determining the tasks and their durations.
- **Step 2** : Determining the sequence of the different operations and identifying the tasks that can be carried out at the same time.
- **Step 3** : Representation in the form of an oriented graph.

Principle

Task or operation : it advances a work towards its final state.

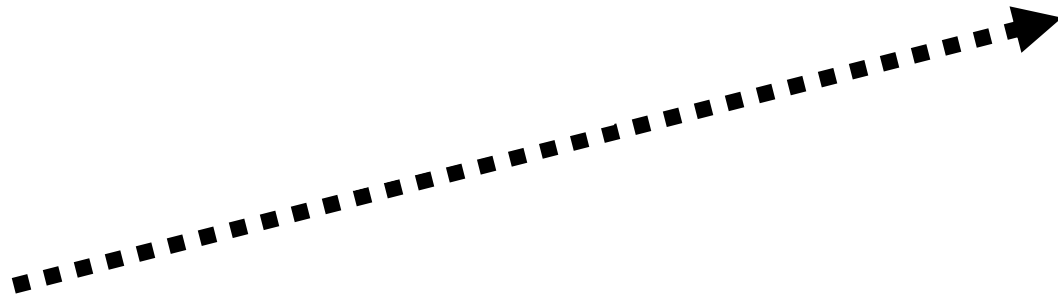
Example of representation of task A of duration 5.



Principle

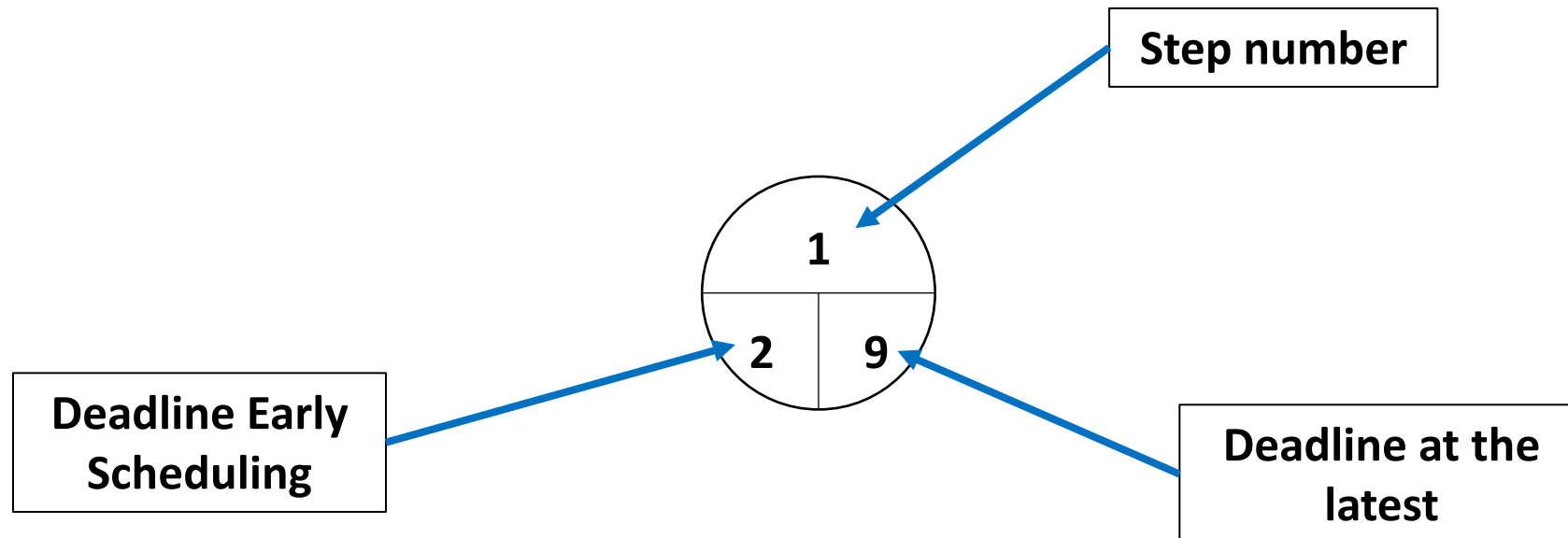
Dummy task (activity):

- A dummy task is a representation of a constraint between non-independent tasks.
- Each fictitious task is represented by a dotted arrow, with a duration of zero.
- Such a task does not consume any resources, and therefore incurs no cost.



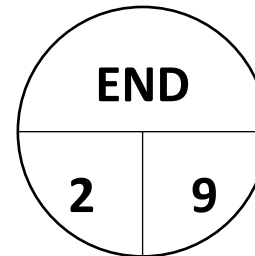
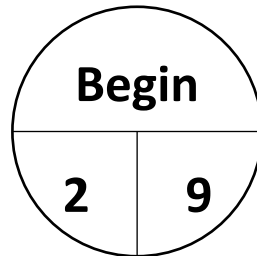
Principle

- A **step** represents the start or end point of a task.
- The following example illustrates step 1.
- Typically, the steps are assigned a numerical designation.
- Furthermore, the earliest and latest completion times are indicated.



Principle

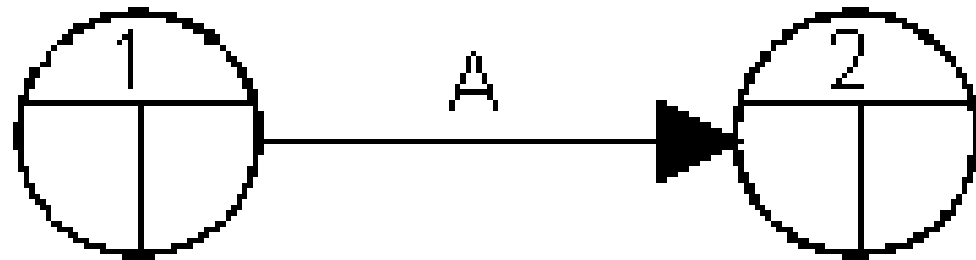
- A **PERT graph** represents the structure of a project, indicating the tasks and steps that constitute the project's.
- It should be noted that :
 - The graph always contains two nodes : a **Begin** step and an **End** step.
 - The graph should be read from left to right.
 - The arrows are pointing this way; there is no possibility of reversal.
 - A task may only be represented by a single arrow.



Rules

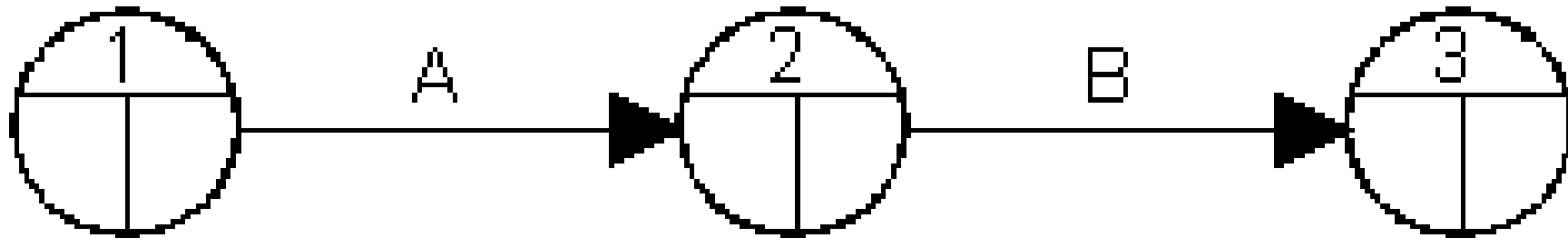
Rule 1

- Every task has a **start** step and an **end** step.
- A next task can only start if the previous task is completed.



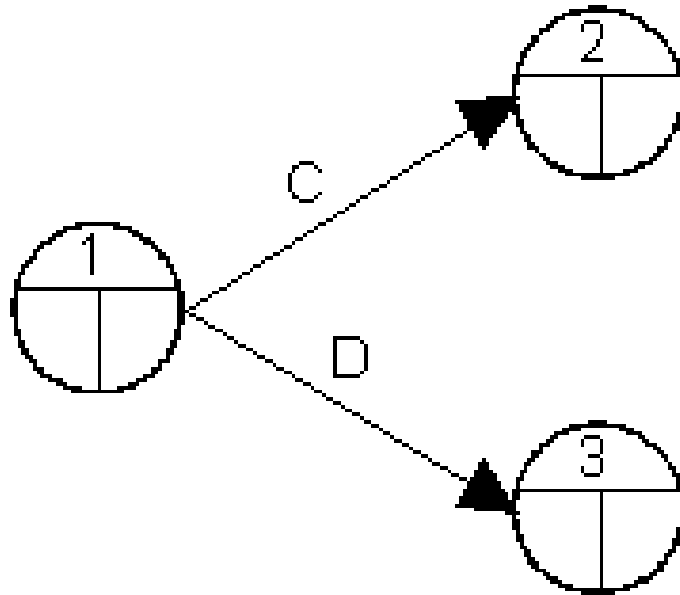
Rules

Rule 2: Two tasks that are consecutive in time are represented by arrows that are consecutive in form.



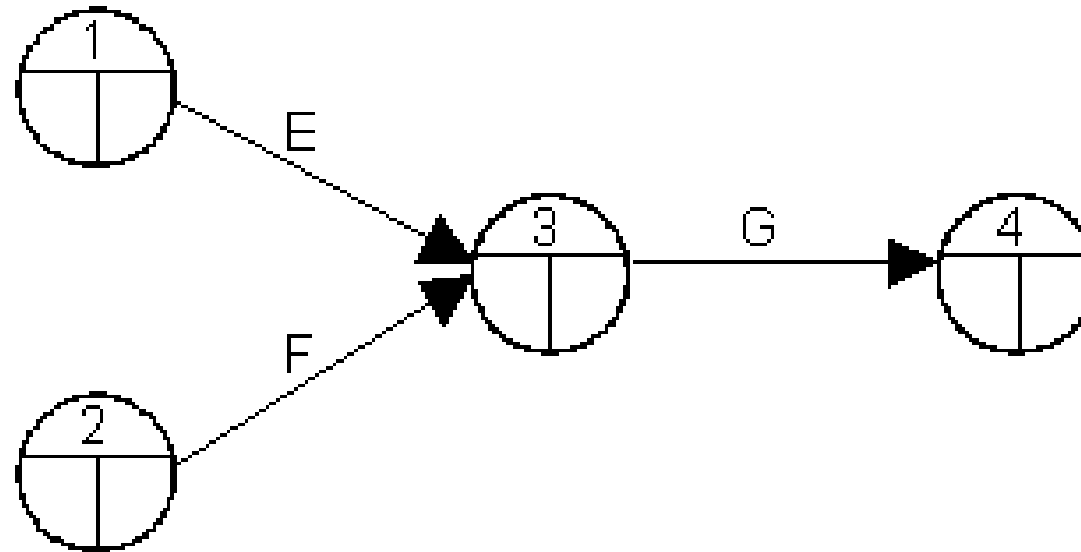
Rules

- Rule 3: Two tasks C and D which are simultaneous are represented as follows:



Rules

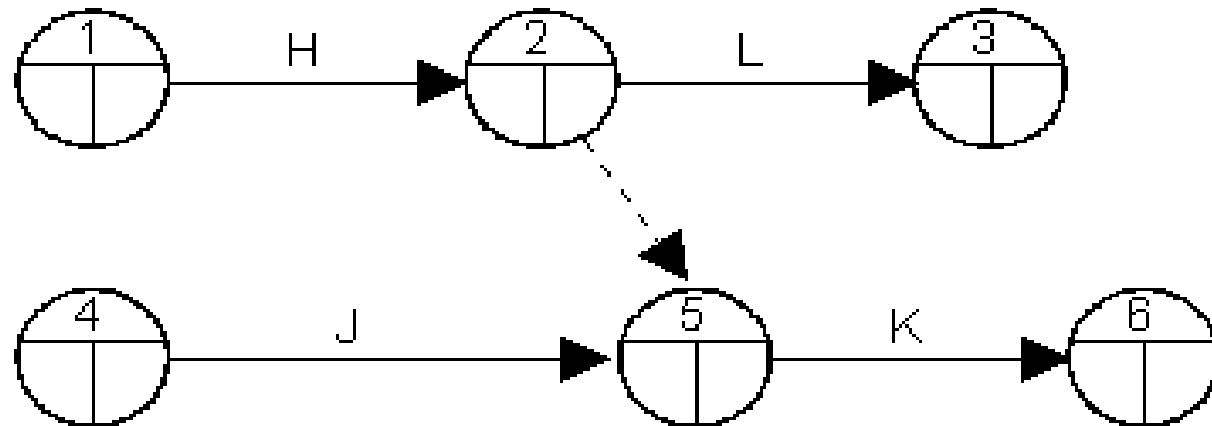
Rule 4: Two tasks E and F which are convergent (i.e., which precede the same task, designated as G), are represented in the following manner:



Rules

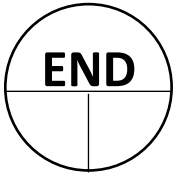
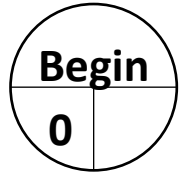
- Rule 5 : In certain instances, it may be appropriate to introduce dummy tasks (fictitious tasks).
- A dummy task is of no consequence in terms of time and does not affect the ultimate deadline.

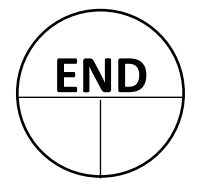
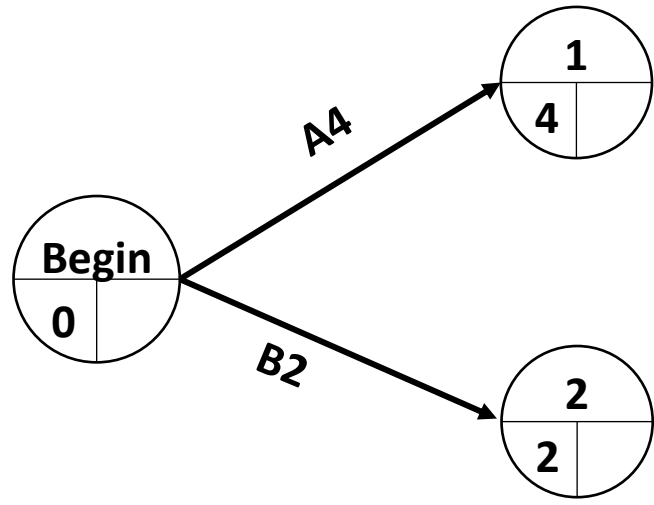
For example, if task K follows tasks H and J, and task L follows only task H, we represent the problem as follows:

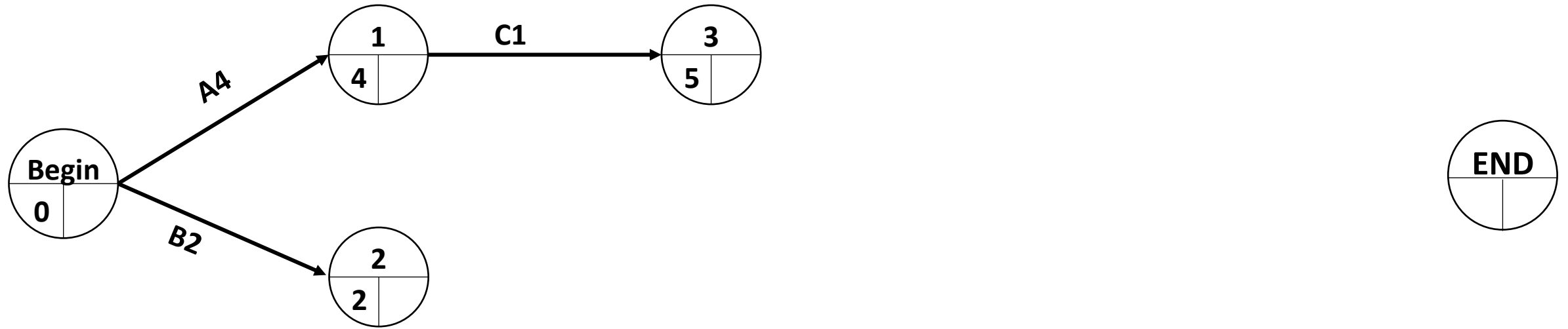


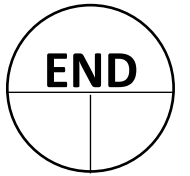
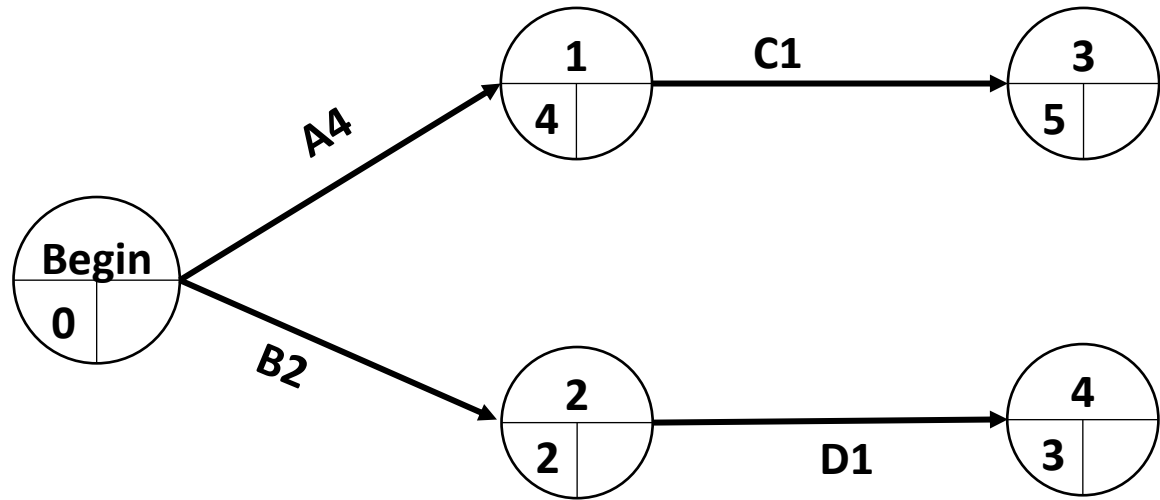
Give PERT chart

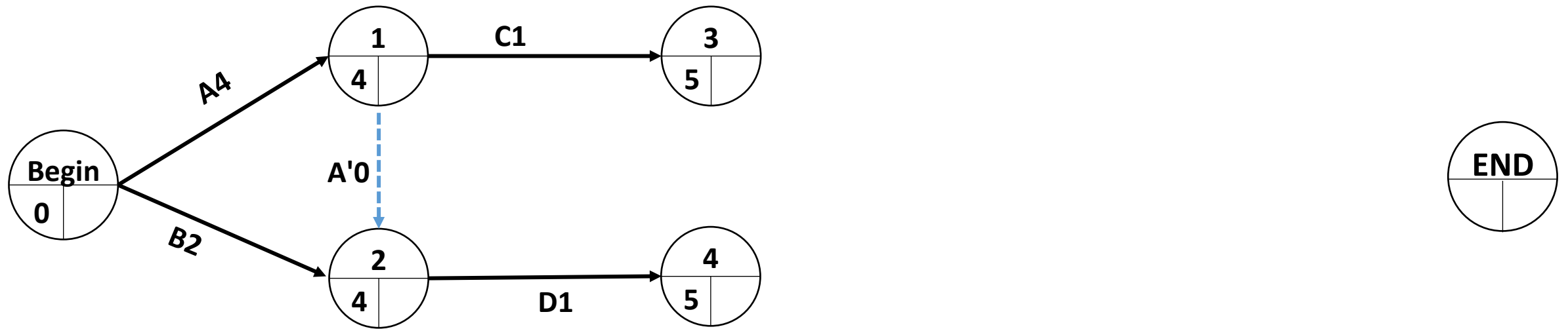
Tasks	Nature	Precedents	Duration (days)
A	Acceptance of Plans by the Owner	-	4
B	Preparing the ground	-	2
C	Ordering materials	A	1
D	Digging the foundations	A,B	1
E	Door and window control	A	2
F	Delivery of materials	C	2
G	Pouring the foundations	D,F	2
H	Delivery of doors and windows	E	10
I	Laying the walls and the framework	G	4
I	Installing the doors	H,I	1

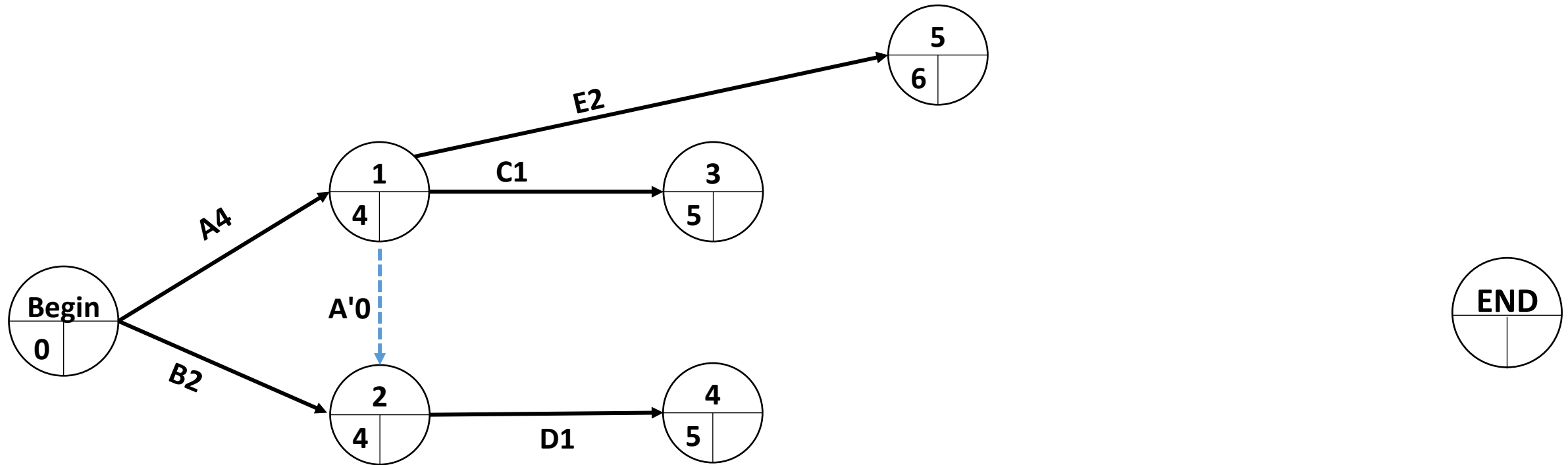


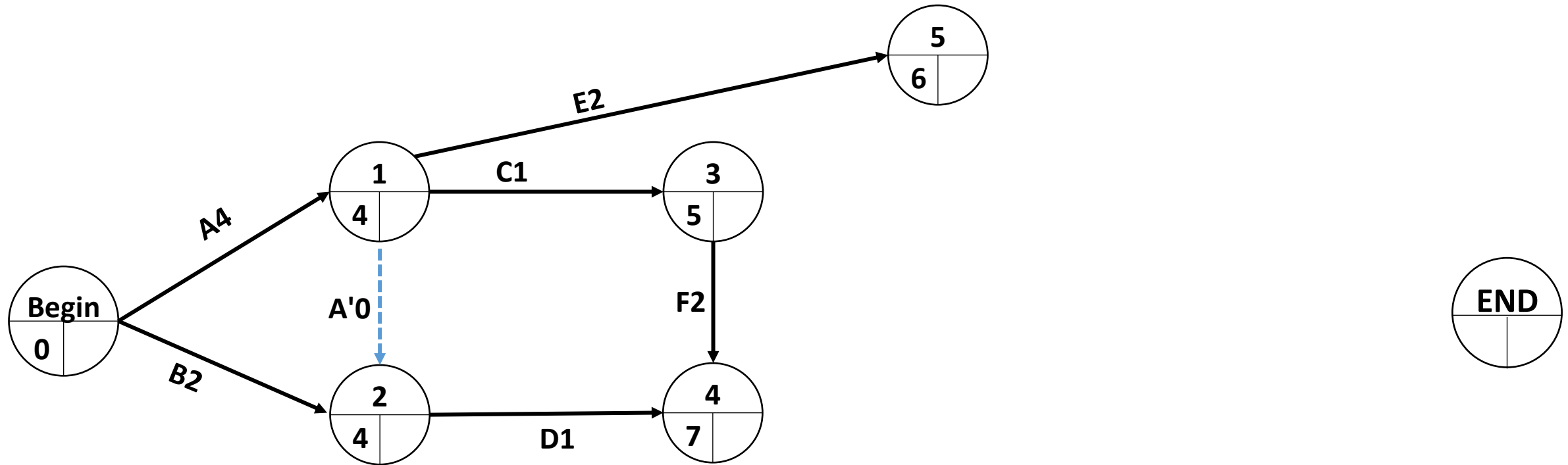


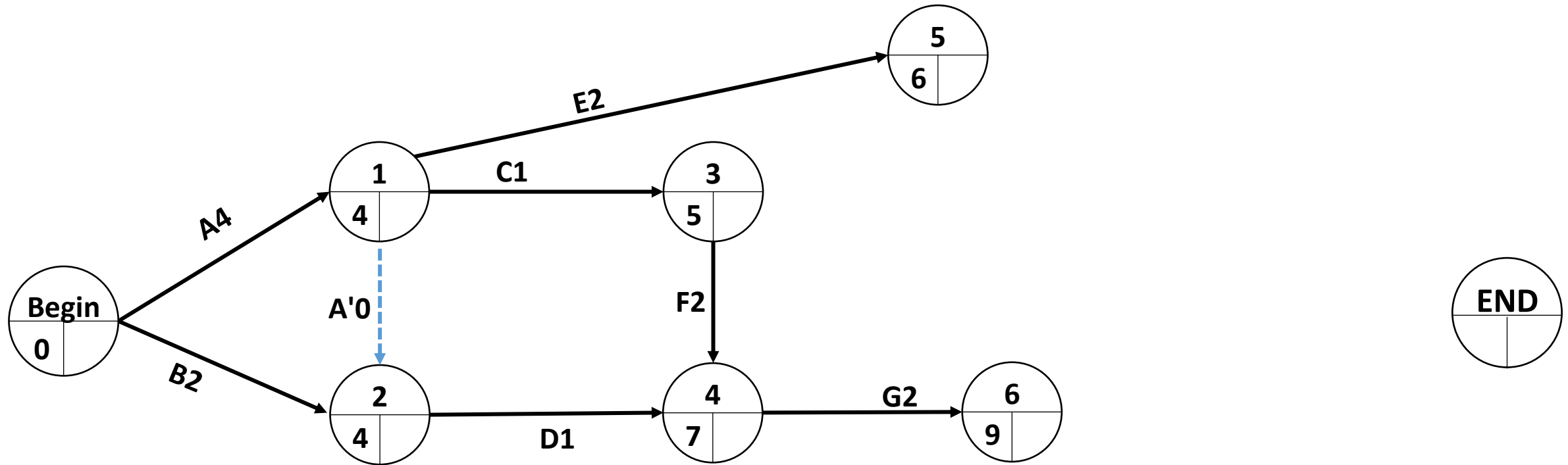


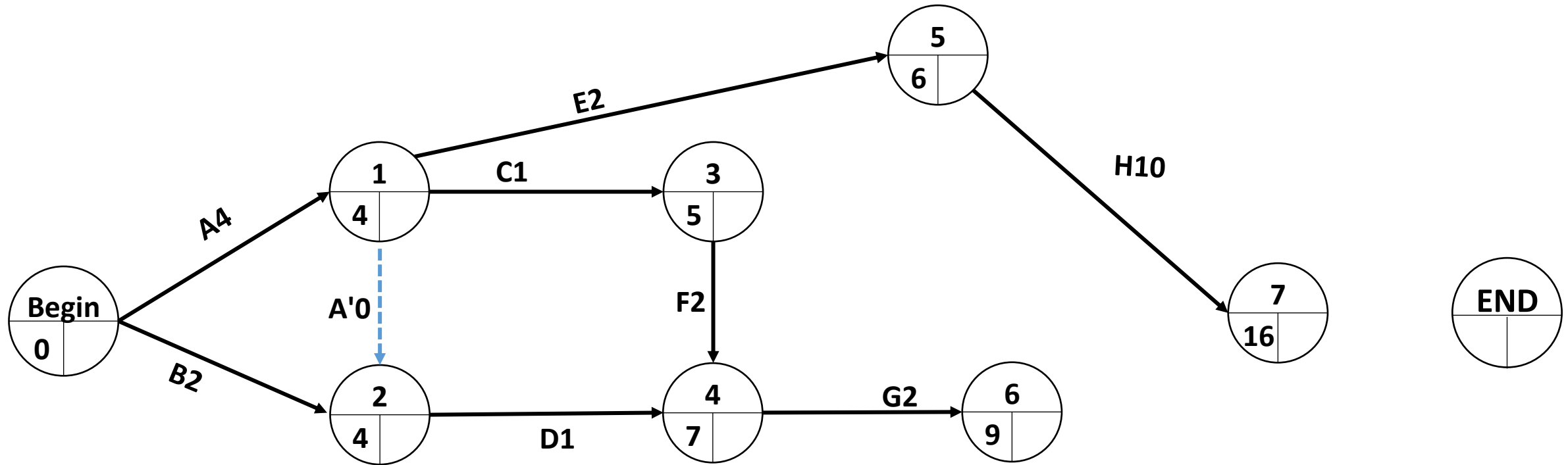


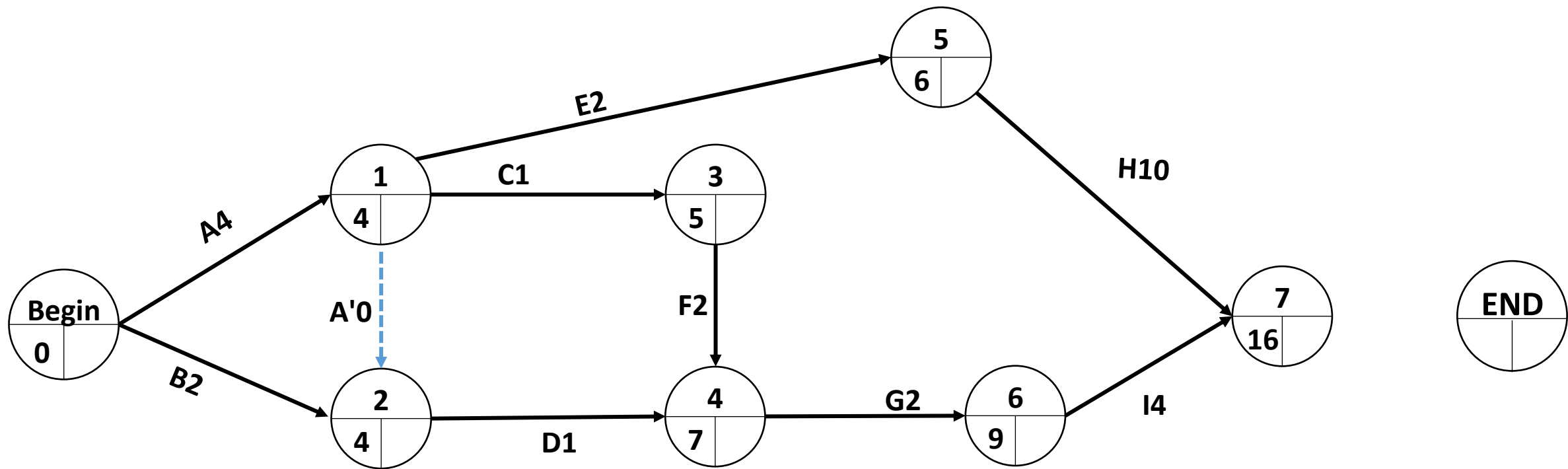


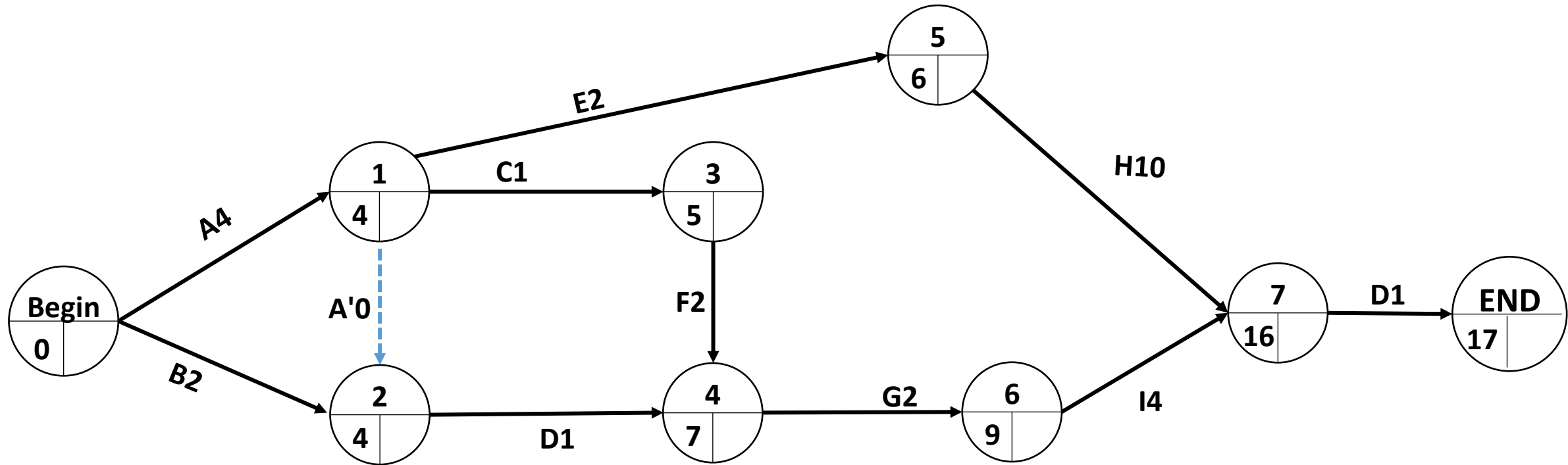


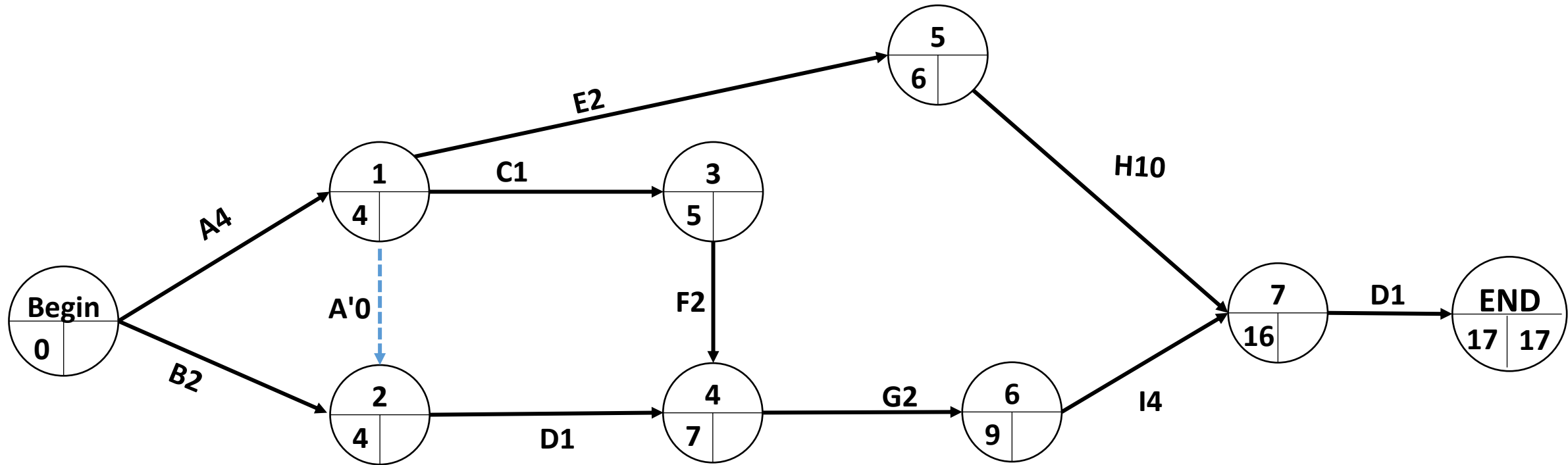


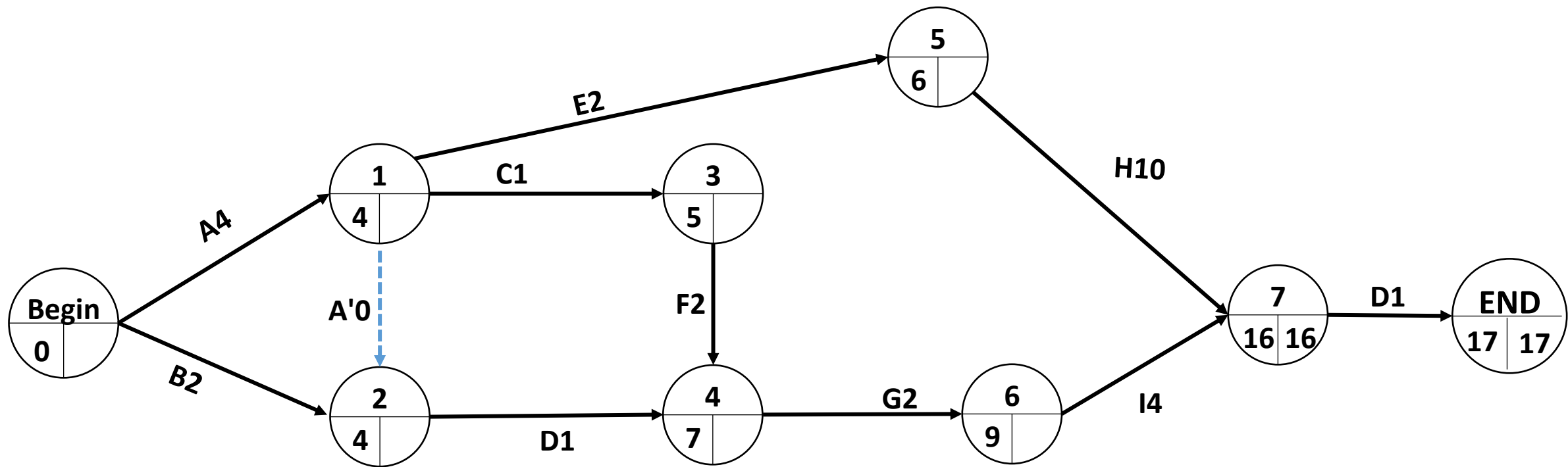


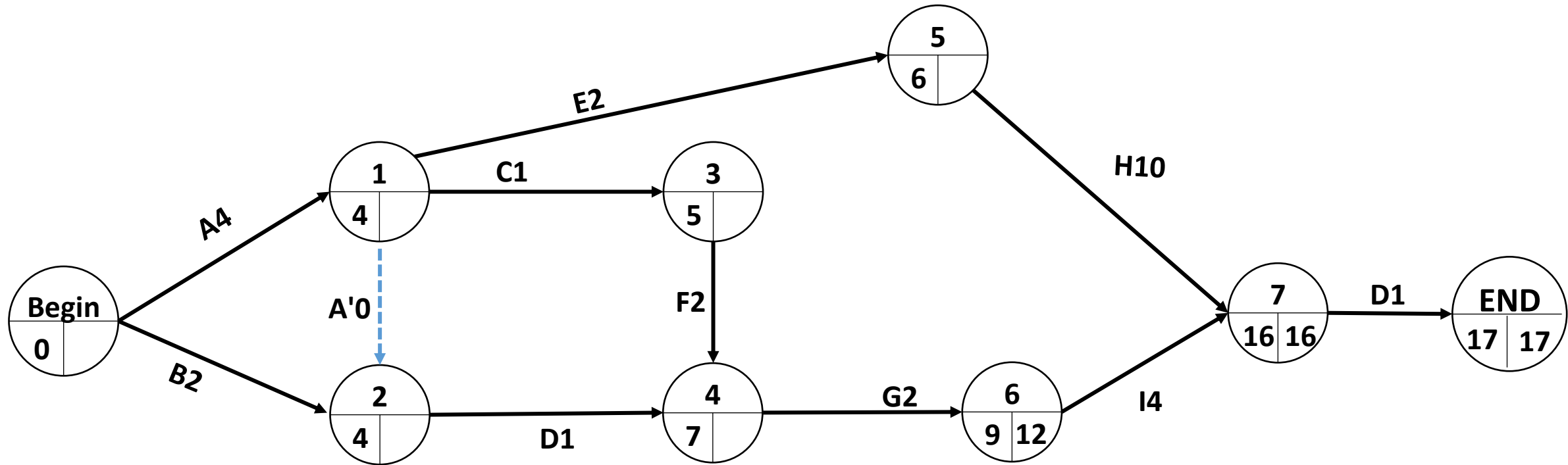


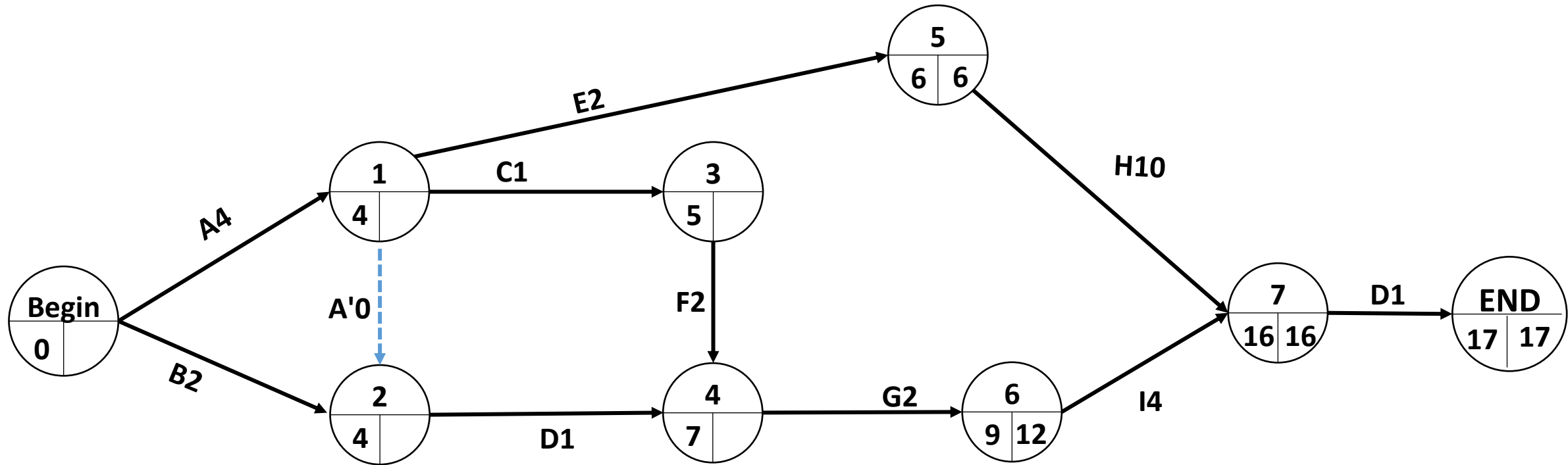


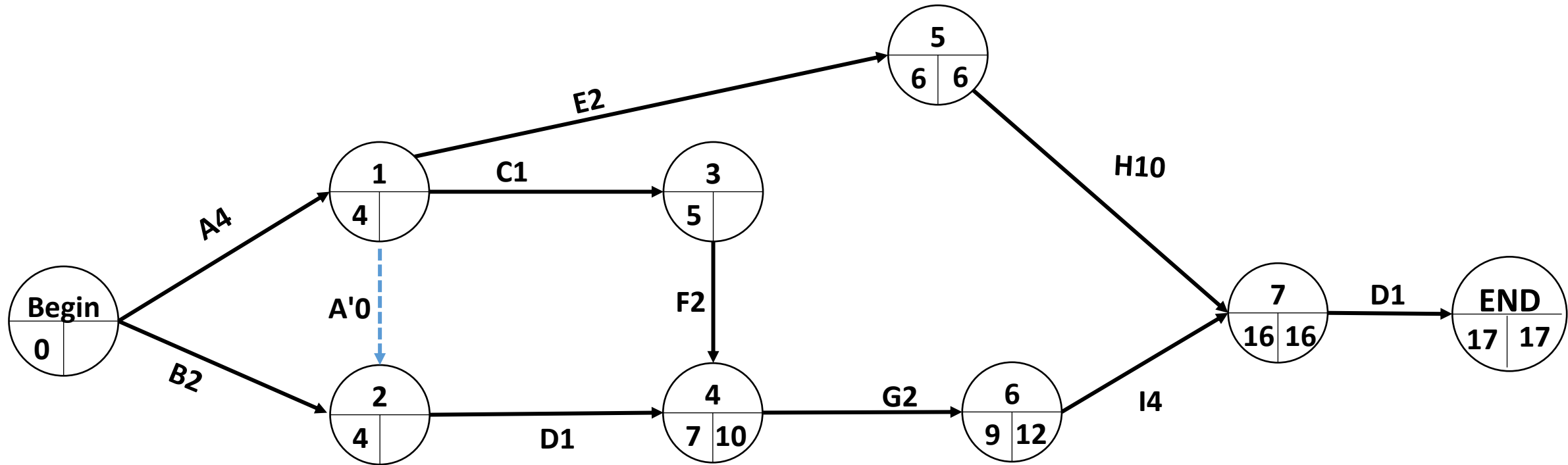


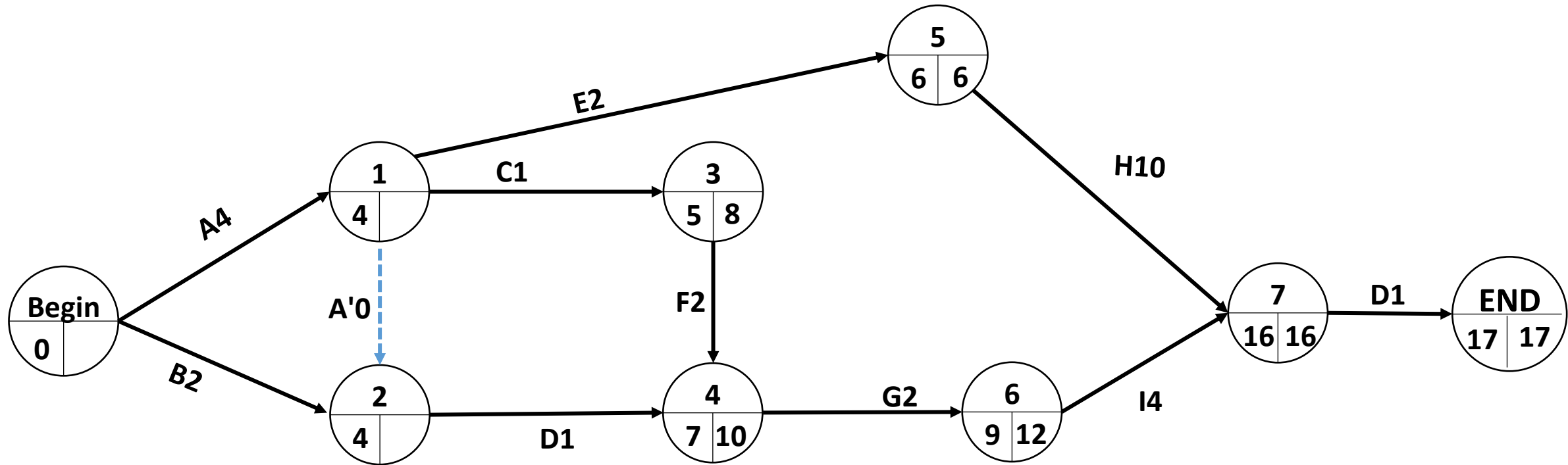


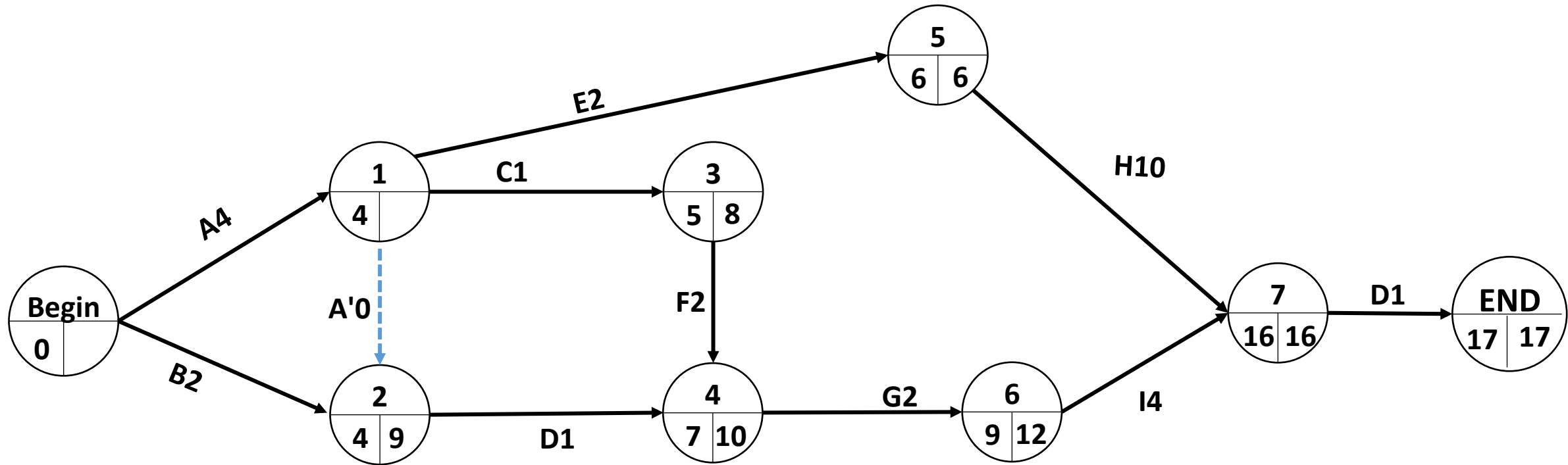


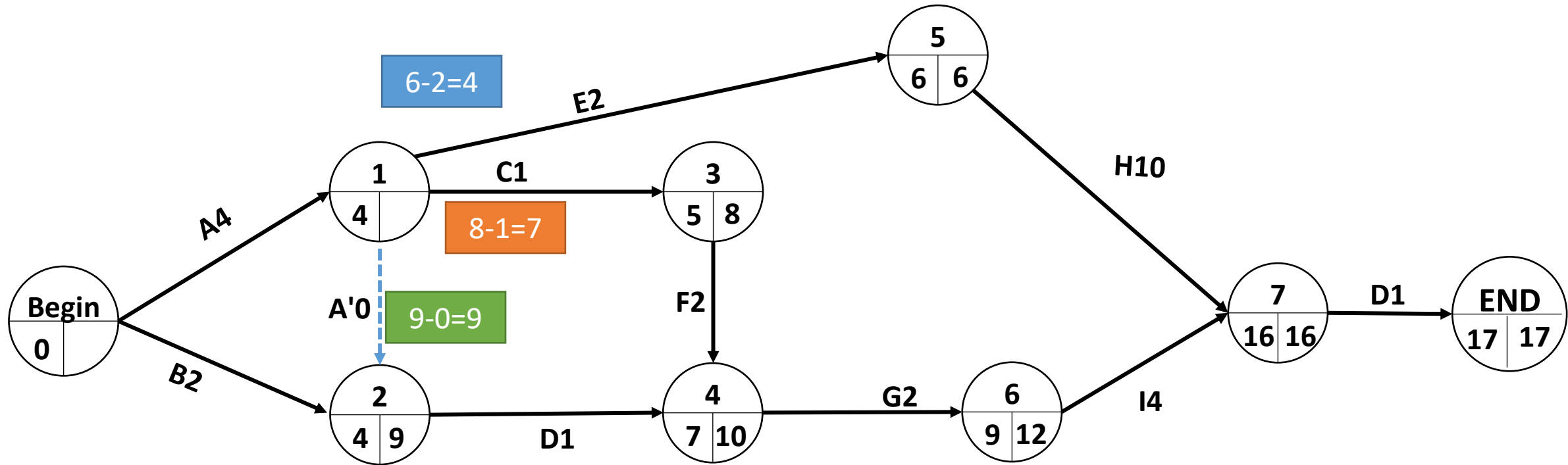


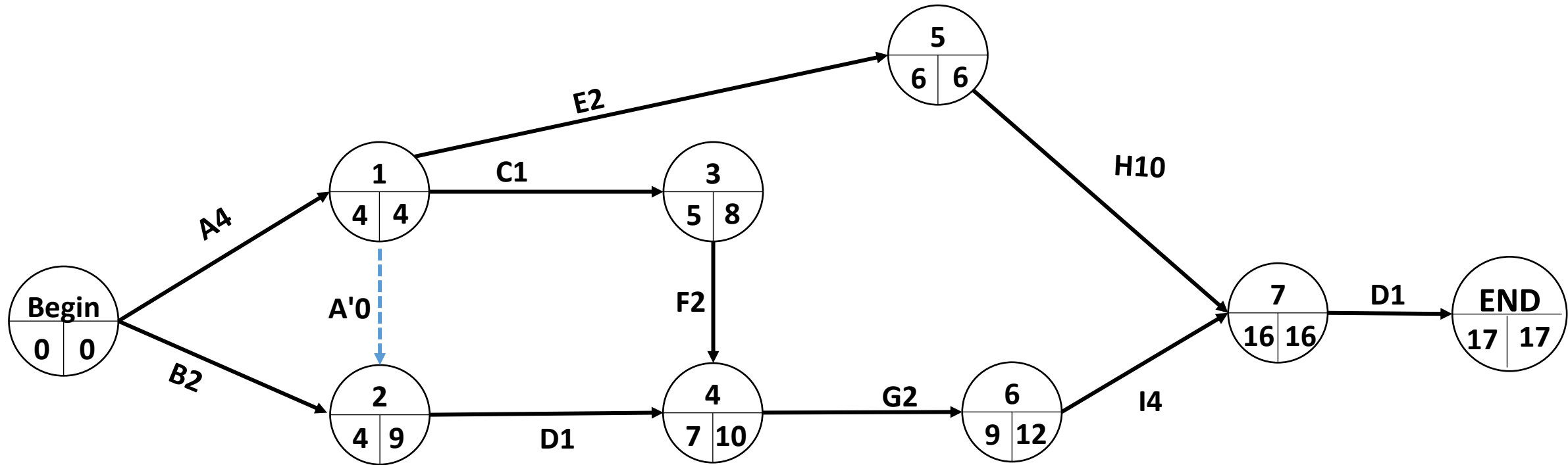


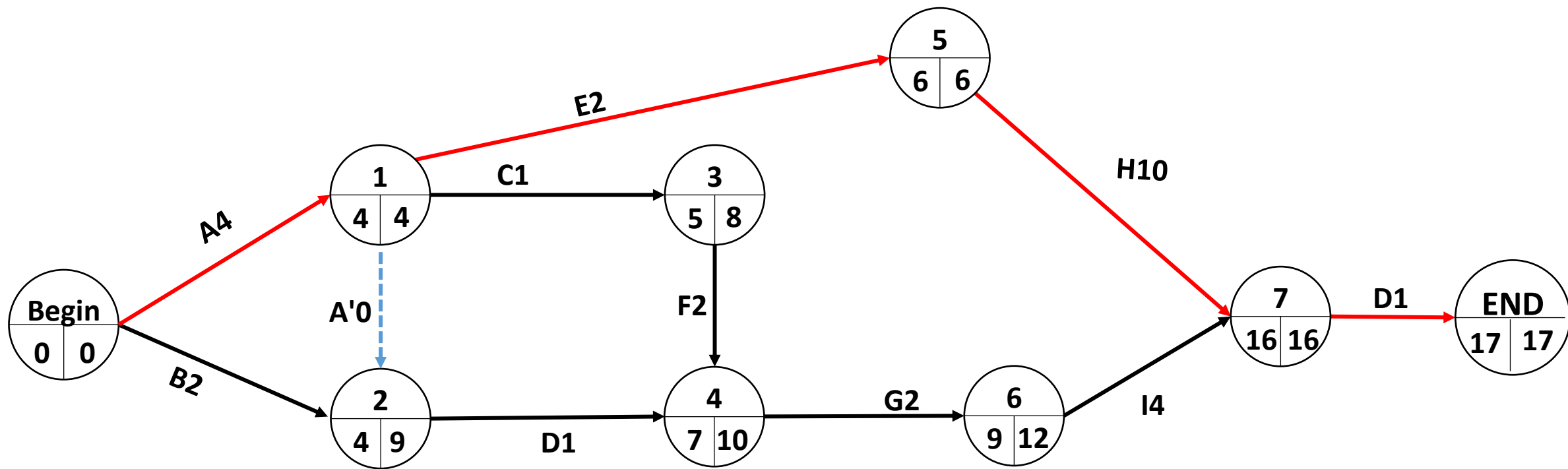








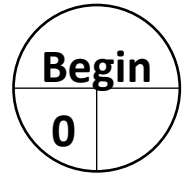




Aspect	Gantt Chart	PERT Method
Purpose	Visualize the schedule and progress of tasks.	Model task dependencies and identify the critical path.
Representation	Bar chart with timelines for each task.	Directed graph with nodes for events and edges for tasks.
Focus	Task durations and sequencing.	Task dependencies and time optimization.
Ease of Use	Easy to create and interpret.	Requires more effort to construct and analyze.
Best For	Projects with straightforward tasks and timelines.	Complex projects with multiple interdependencies.
Analysis	Tracks progress but doesn't explicitly identify critical tasks.	Highlights critical tasks and slack for optimization.
Flexibility	Less flexible for real-time adjustments.	Adaptable to changes in project tasks or dependencies.
Visualization	Provides a clear, intuitive overview.	Offers detailed insights but can be harder to visualize for large projects.
Software Support	Widely supported in basic tools (e.g., Excel, MS Project).	Often requires specialized project management software.

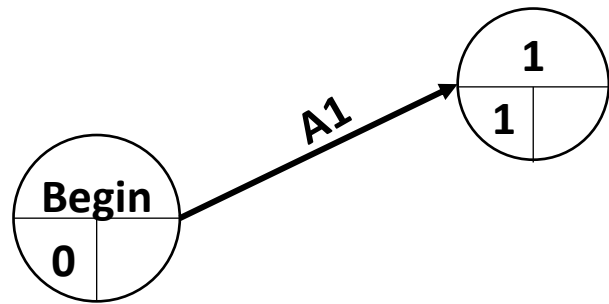
Example:

Tasks	A	B	C	D	E	F	G	H	I	I	K	L	M
Duration	1	2	1	3	2	5	2	5	2	1	4	5	4
Previous works	-	-	A	-	B	E	CD	-	H	-	I,J	F,G	K,L



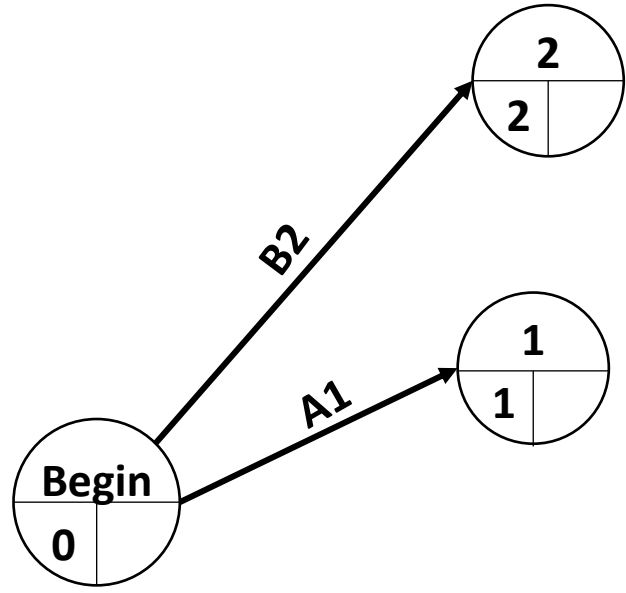
Example:

Tasks	A	B	C	D	E	F	G	H	I	I	K	L	M
Duration	1	2	1	3	2	5	2	5	2	1	4	5	4
Previous works	-	-	A	-	B	E	CD	-	H	-	I,J	F,G	K,L



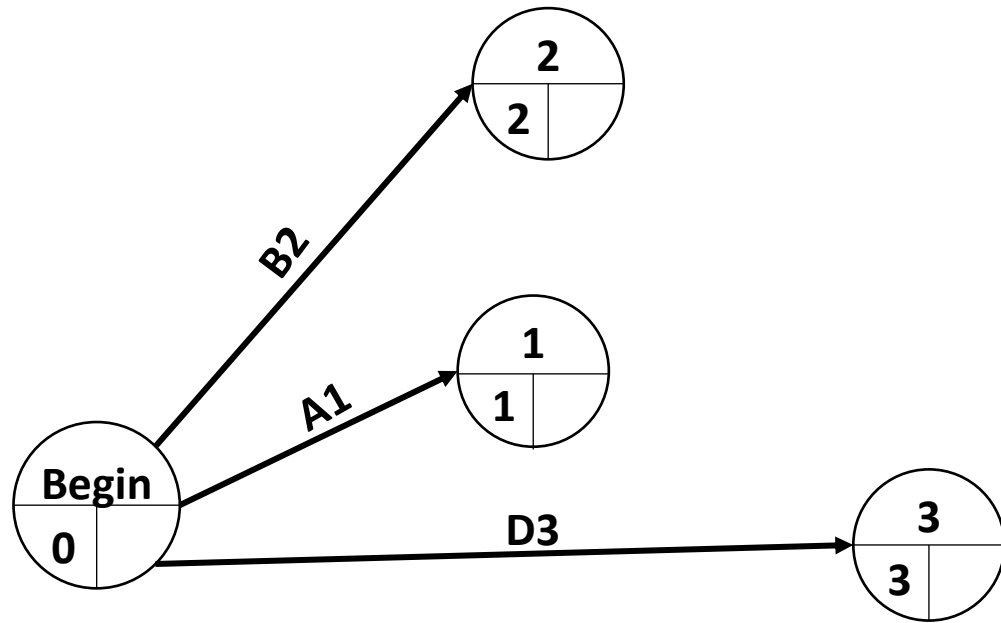
Example:

Tasks	A	B	C	D	E	F	G	H	I	I	K	L	M
Duration	1	2	1	3	2	5	2	5	2	1	4	5	4
Previous works	-	-	A	-	B	E	CD	-	H	-	I,J	F,G	K,L



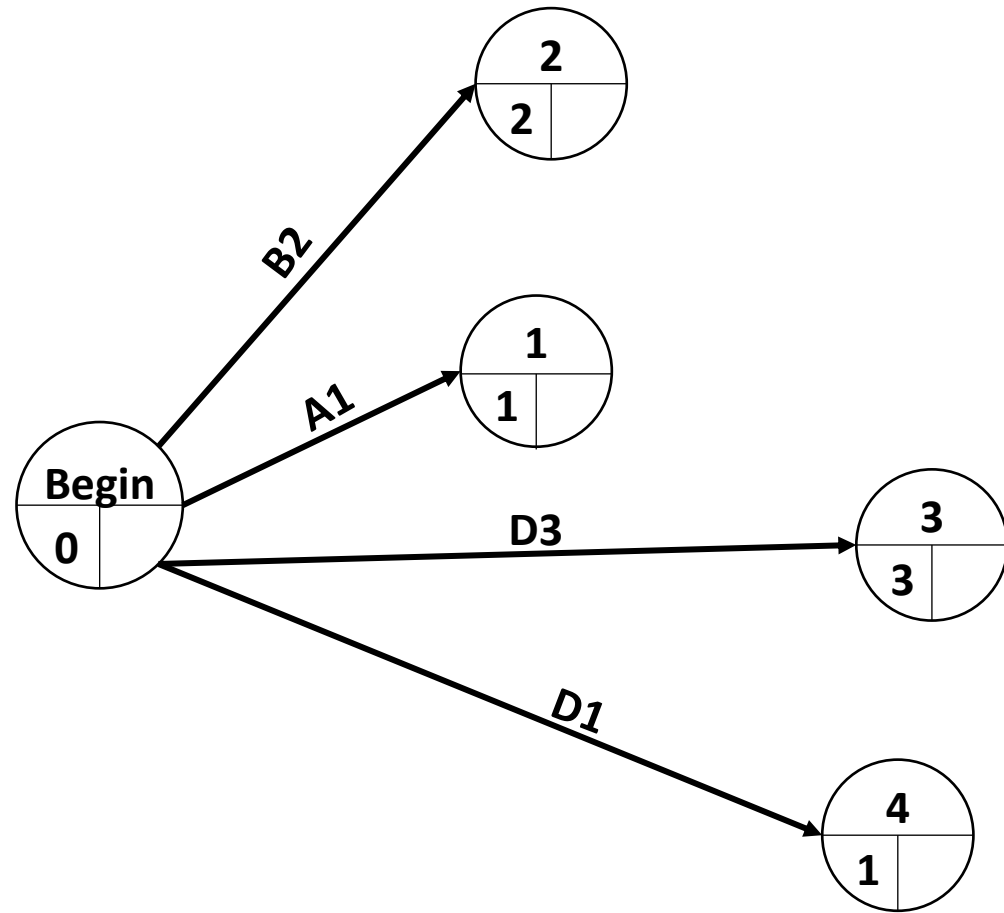
Example:

Tasks	A	B	C	D	E	F	G	H	I	I	K	L	M
Duration	1	2	1	3	2	5	2	5	2	1	4	5	4
Previous works	-	-	A	-	B	E	CD	-	H	-	I,J	F,G	K,L



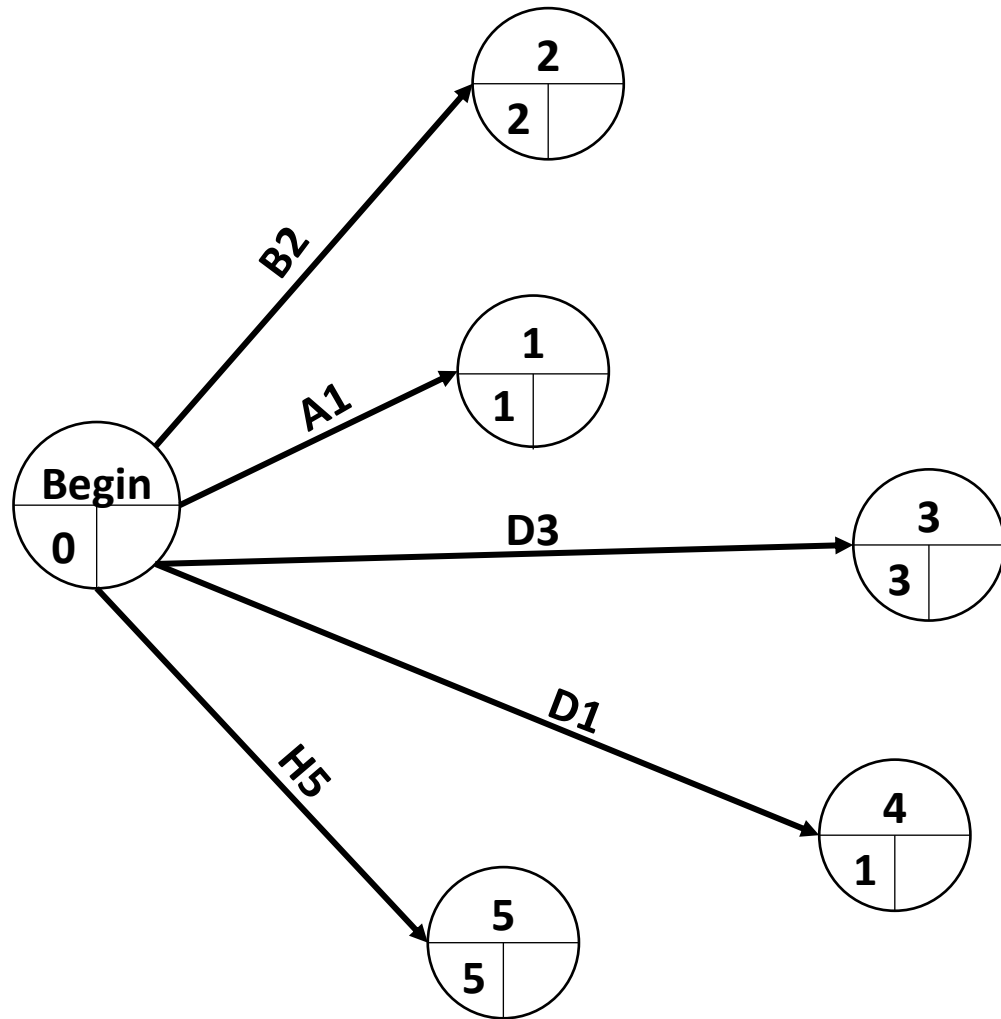
Example:

Tasks	A	B	C	D	E	F	G	H	I	I	K	L	M
Duration	1	2	1	3	2	5	2	5	2	1	4	5	4
Previous works	-	-	A	-	B	E	CD	-	H	-	I,J	F,G	K,L



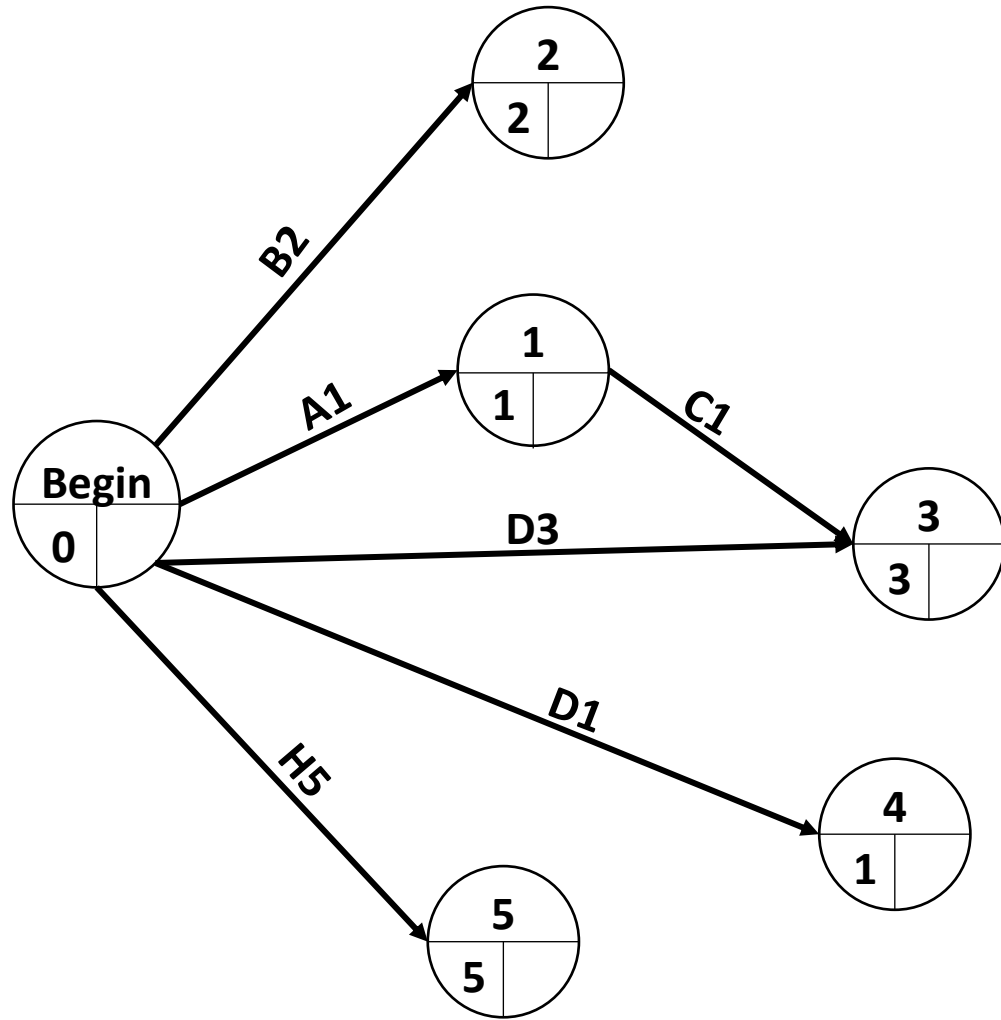
Example:

Tasks	A	B	C	D	E	F	G	H	I	I	K	L	M
Duration	1	2	1	3	2	5	2	5	2	1	4	5	4
Previous works	-	-	A	-	B	E	CD	-	H	-	I,J	F,G	K,L



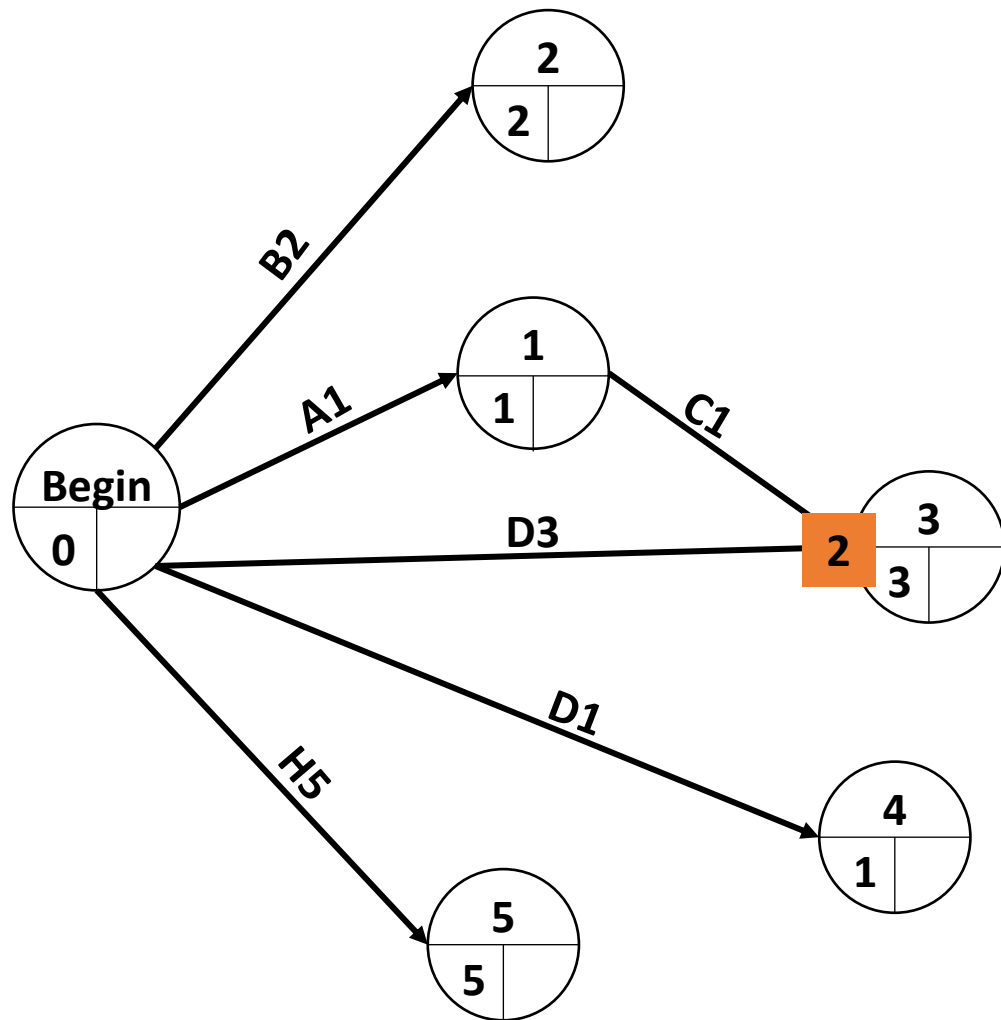
Example:

Tasks	A	B	C	D	E	F	G	H	I	I	K	L	M
Duration	1	2	1	3	2	5	2	5	2	1	4	5	4
Previous works	-	-	A	-	B	E	CD	-	H	-	I,J	F,G	K,L



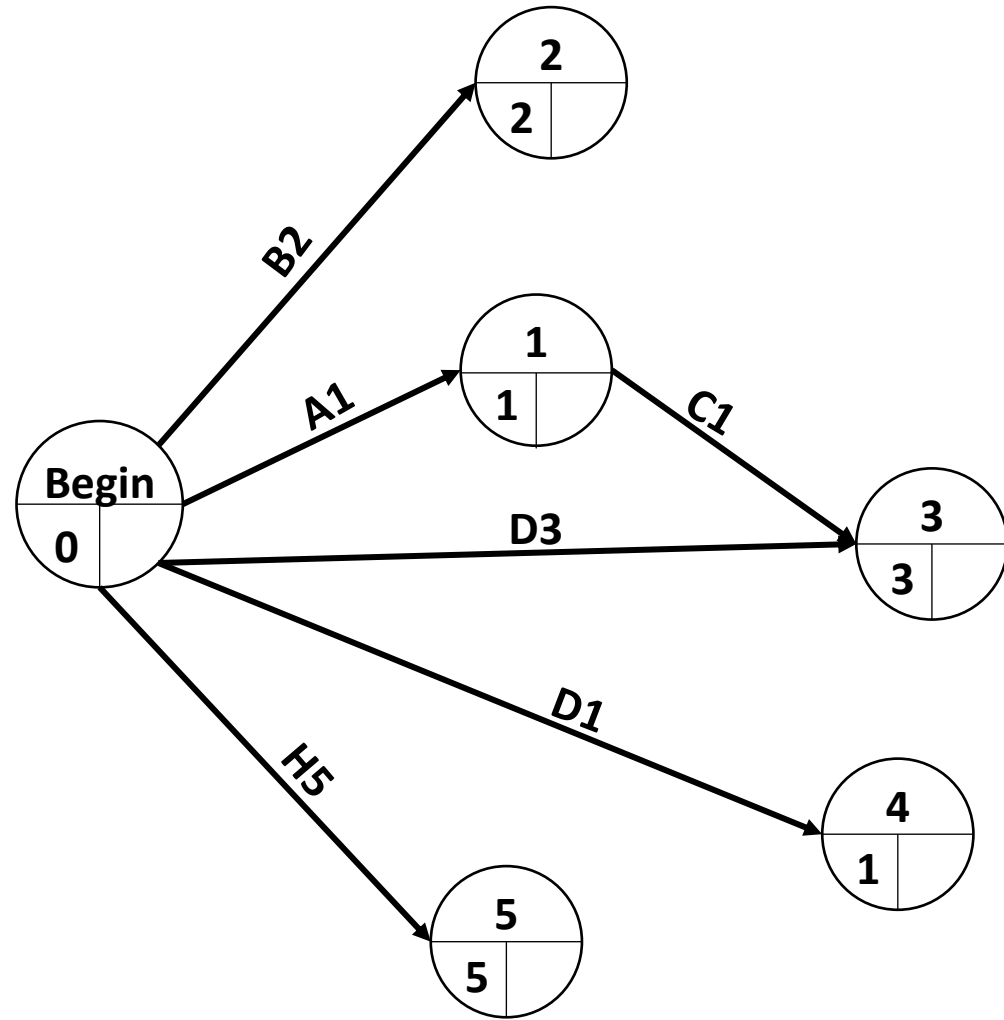
Example:

Tasks	A	B	C	D	E	F	G	H	I	I	K	L	M
Duration	1	2	1	3	2	5	2	5	2	1	4	5	4
Previous works	-	-	A	-	B	E	CD	-	H	-	I,J	F,G	K,L



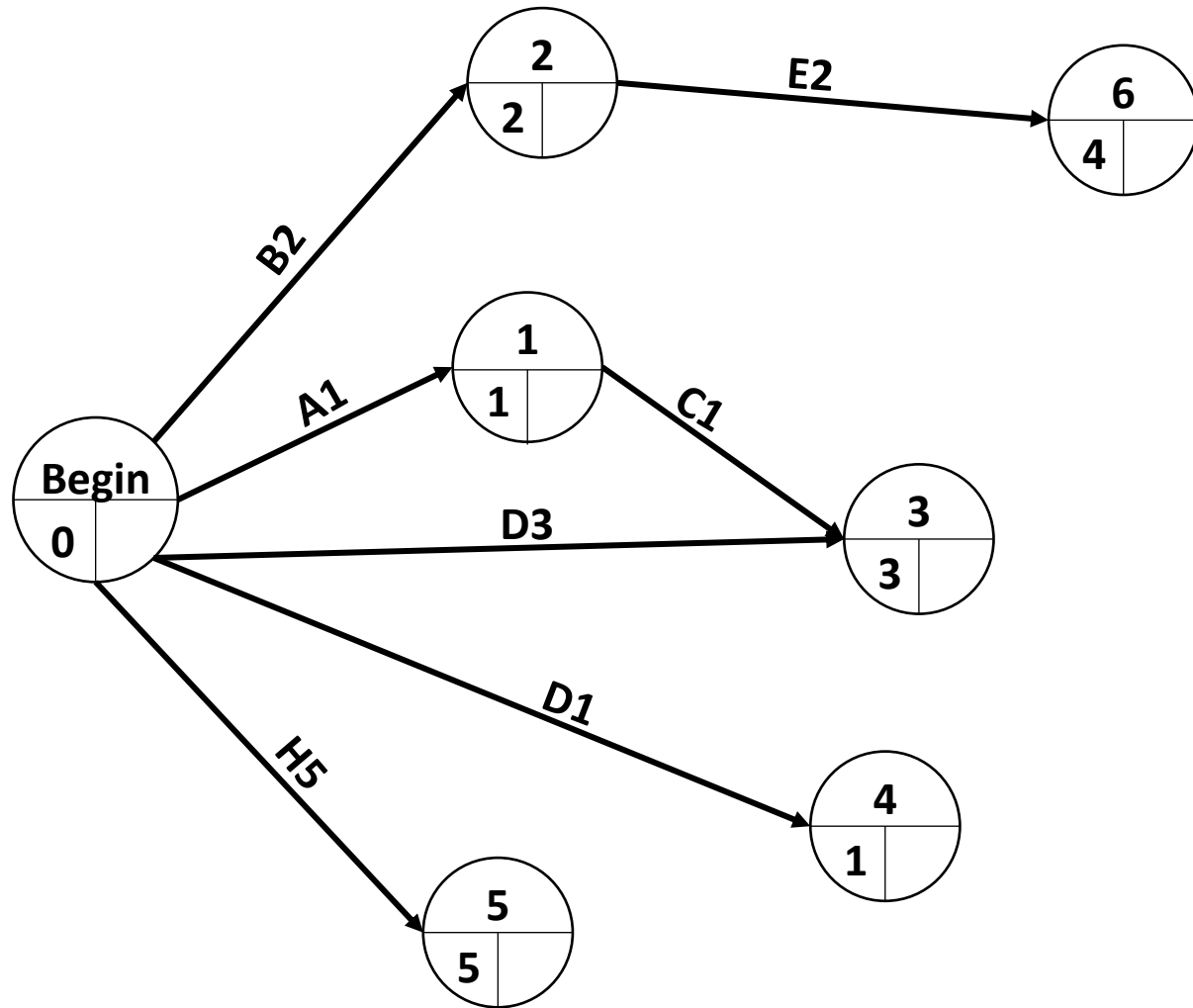
Example:

Tasks	A	B	C	D	E	F	G	H	I	I	K	L	M
Duration	1	2	1	3	2	5	2	5	2	1	4	5	4
Previous works	-	-	A	-	B	E	CD	-	H	-	I,J	F,G	K,L



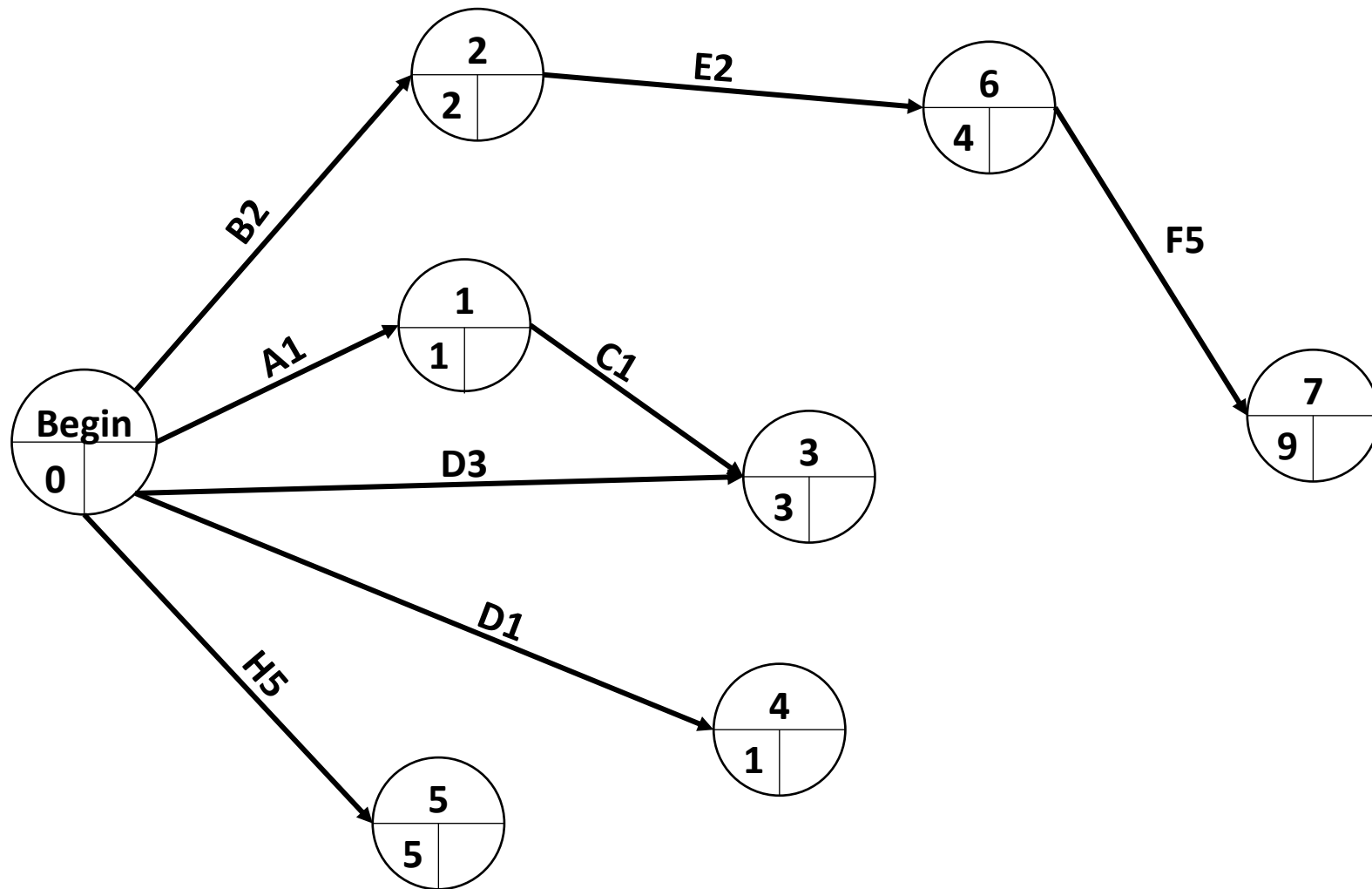
Example:

Tasks	A	B	C	D	E	F	G	H	I	I	K	L	M
Duration	1	2	1	3	2	5	2	5	2	1	4	5	4
Previous works	-	-	A	-	B	E	CD	-	H	-	I,J	F,G	K,L



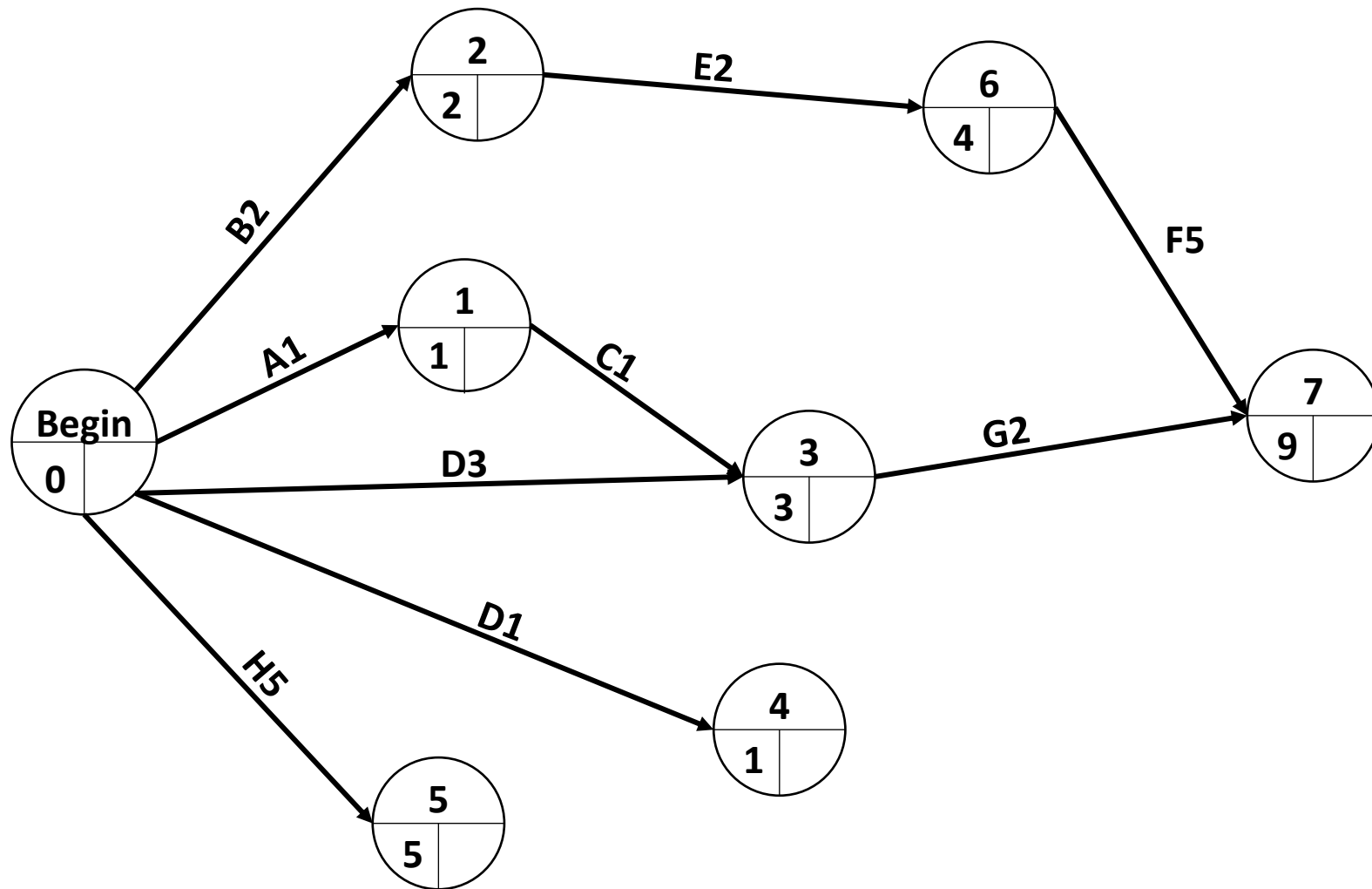
Example:

Tasks	A	B	C	D	E	F	G	H	I	I	K	L	M
Duration	1	2	1	3	2	5	2	5	2	1	4	5	4
Previous works	-	-	A	-	B	E	CD	-	H	-	I,J	F,G	K,L



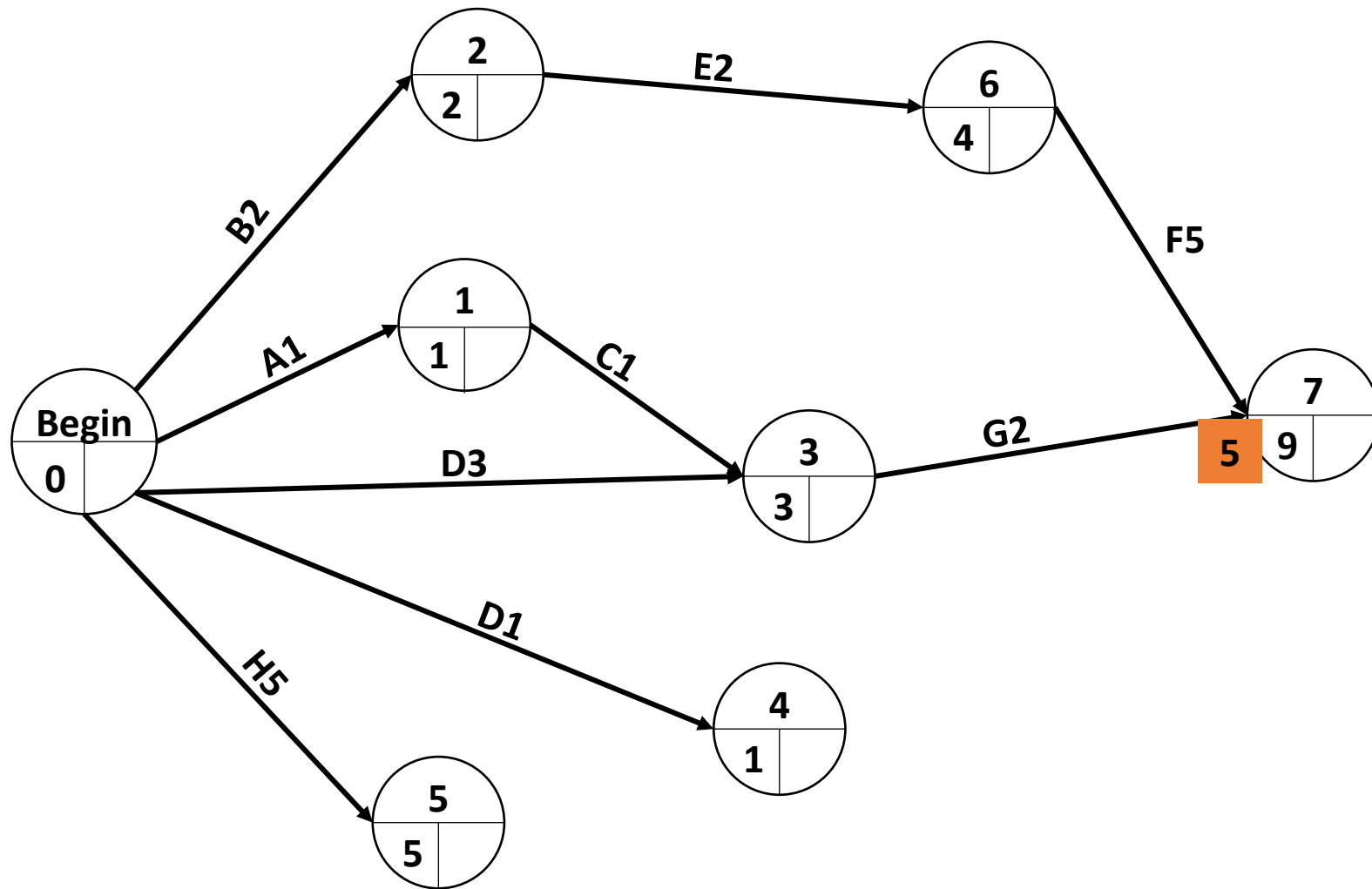
Example:

Tasks	A	B	C	D	E	F	G	H	I	I	K	L	M
Duration	1	2	1	3	2	5	2	5	2	1	4	5	4
Previous works	-	-	A	-	B	E	CD	-	H	-	I,J	F,G	K,L



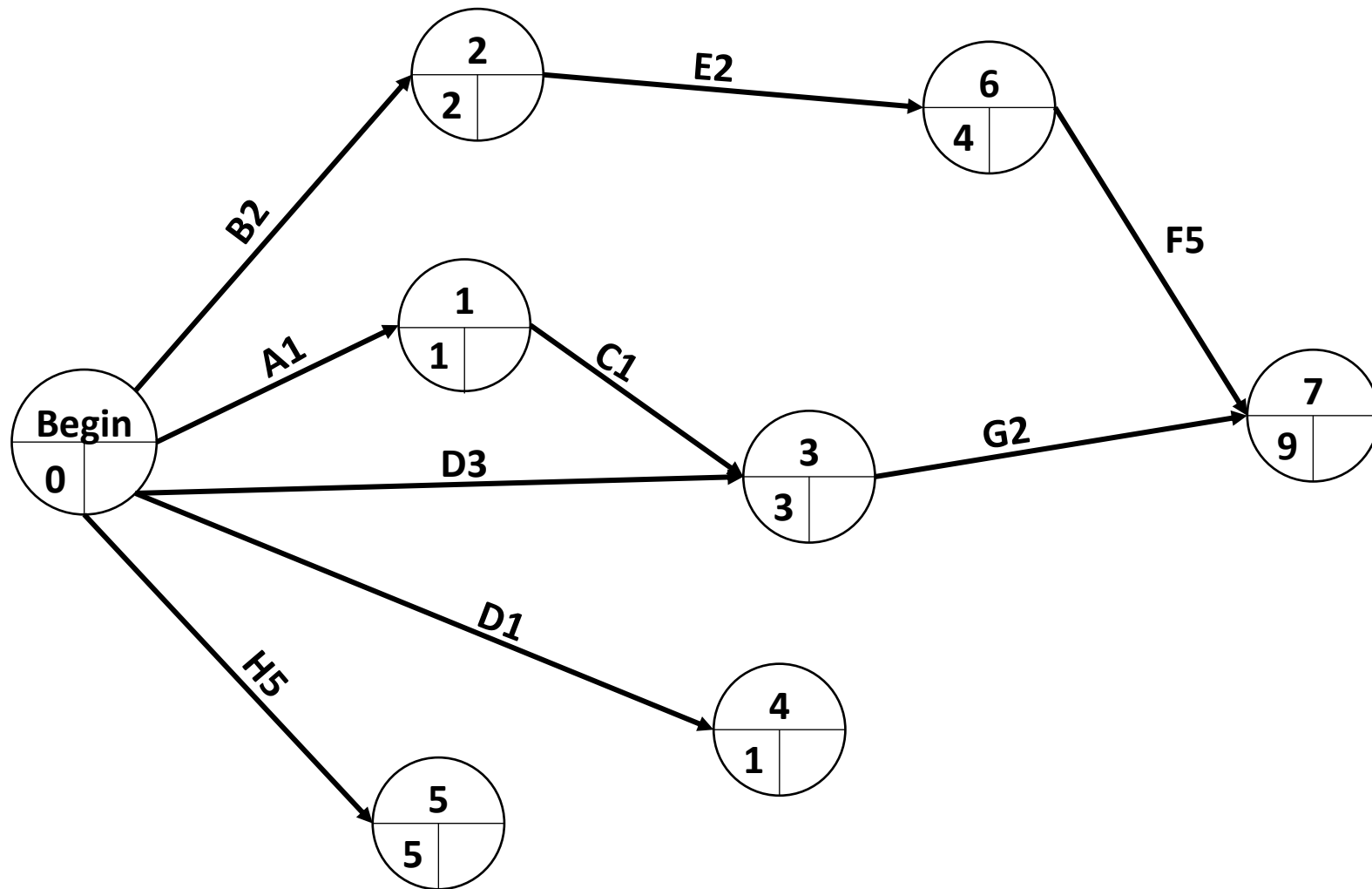
Example:

Tasks	A	B	C	D	E	F	G	H	I	I	K	L	M
Duration	1	2	1	3	2	5	2	5	2	1	4	5	4
Previous works	-	-	A	-	B	E	CD	-	H	-	I,J	F,G	K,L



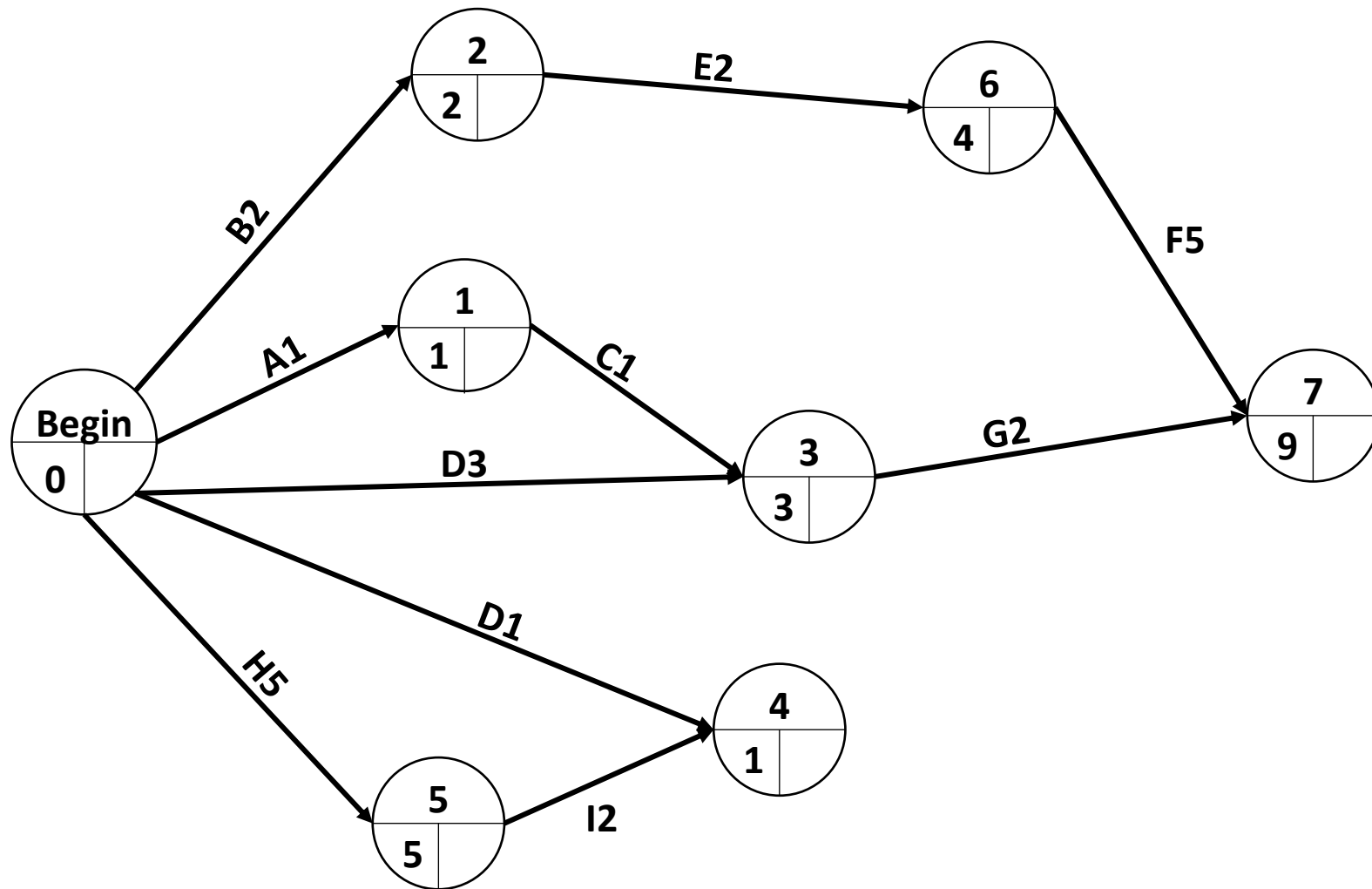
Example:

Tasks	A	B	C	D	E	F	G	H	I	I	K	L	M
Duration	1	2	1	3	2	5	2	5	2	1	4	5	4
Previous works	-	-	A	-	B	E	CD	-	H	-	I,J	F,G	K,L



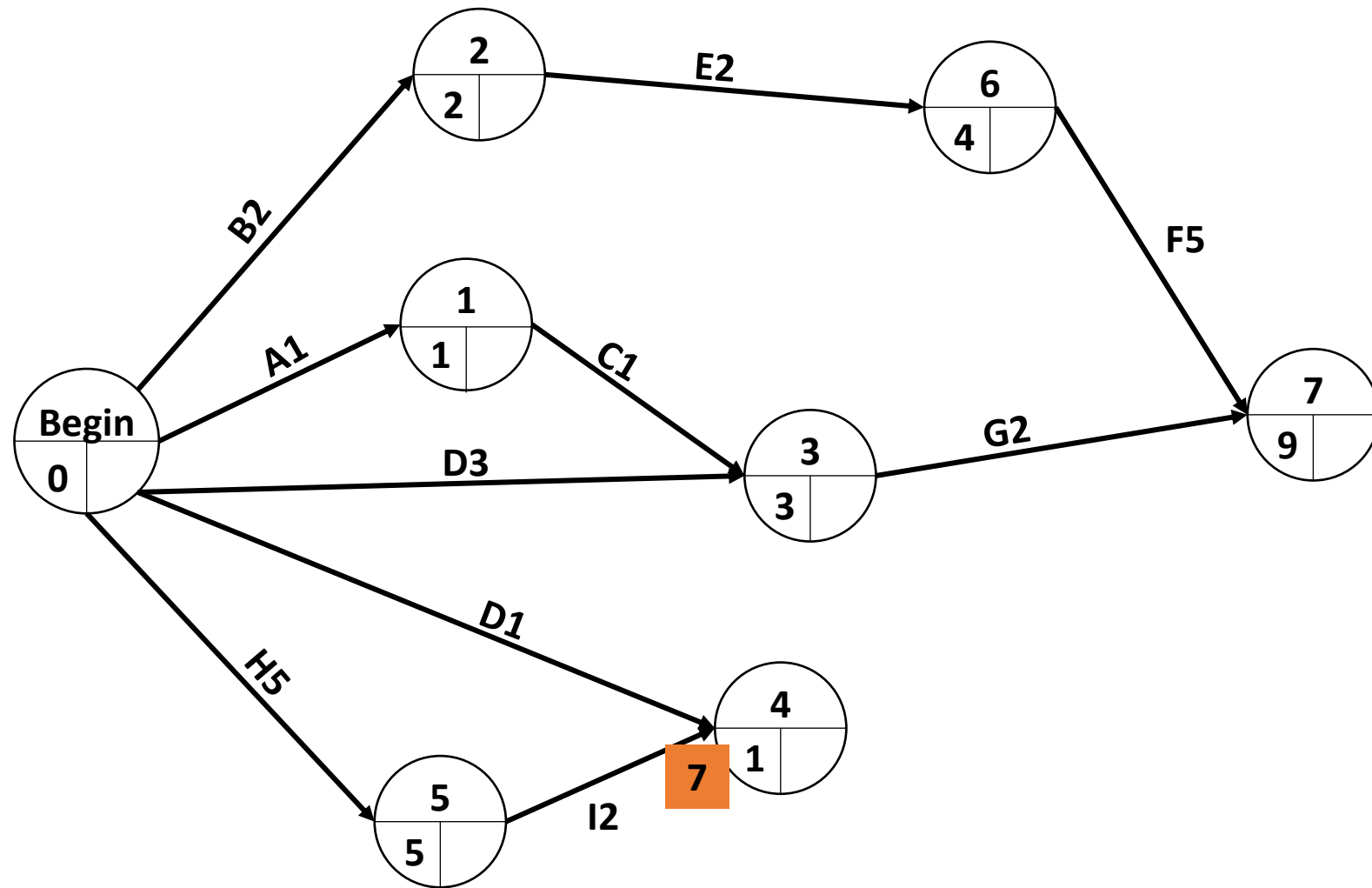
Example:

Tasks	A	B	C	D	E	F	G	H	I	I	K	L	M
Duration	1	2	1	3	2	5	2	5	2	1	4	5	4
Previous works	-	-	A	-	B	E	CD	-	H	-	I,J	F,G	K,L



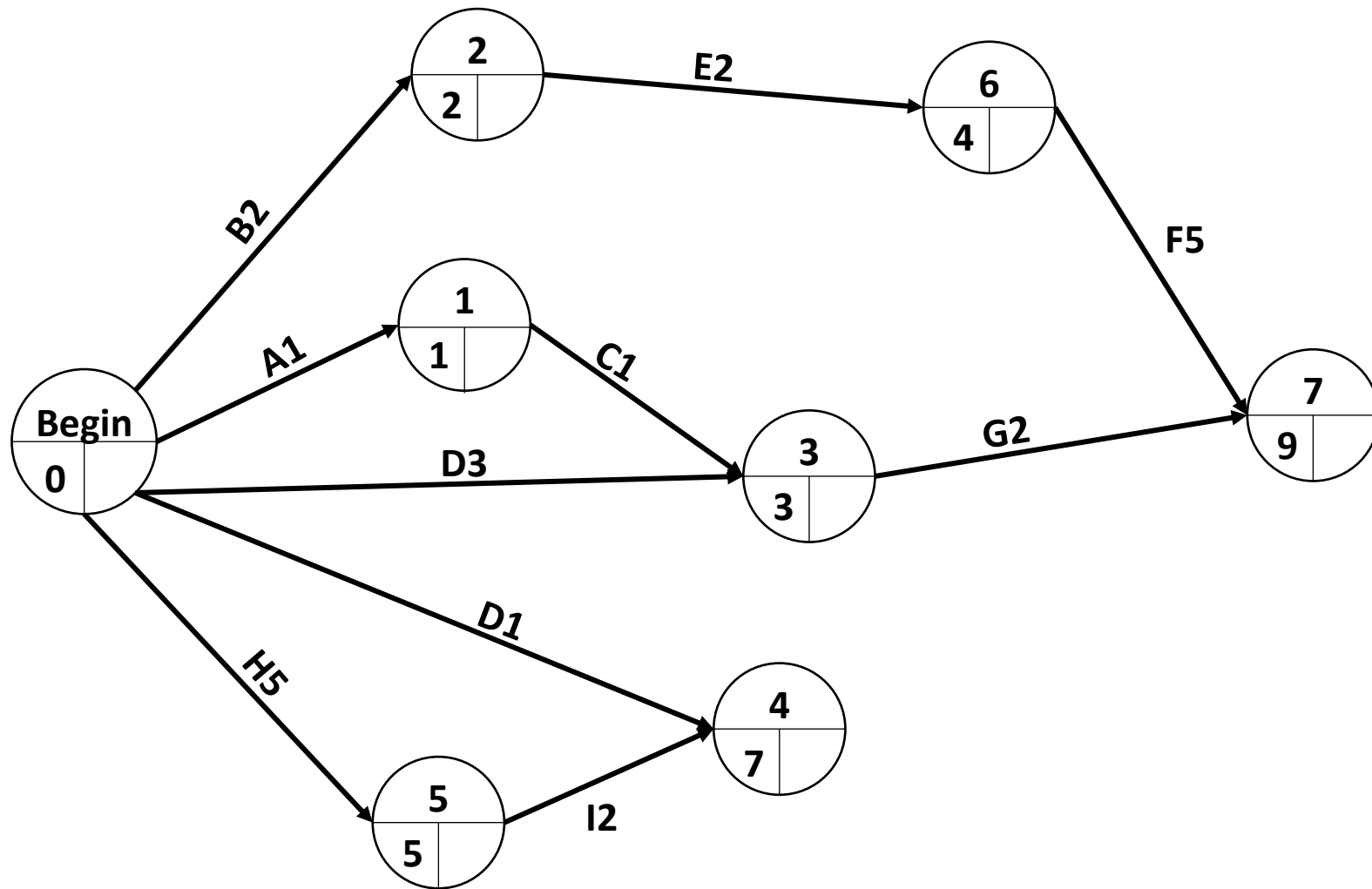
Example:

Tasks	A	B	C	D	E	F	G	H	I	I	K	L	M
Duration	1	2	1	3	2	5	2	5	2	1	4	5	4
Previous works	-	-	A	-	B	E	CD	-	H	-	I,J	F,G	K,L



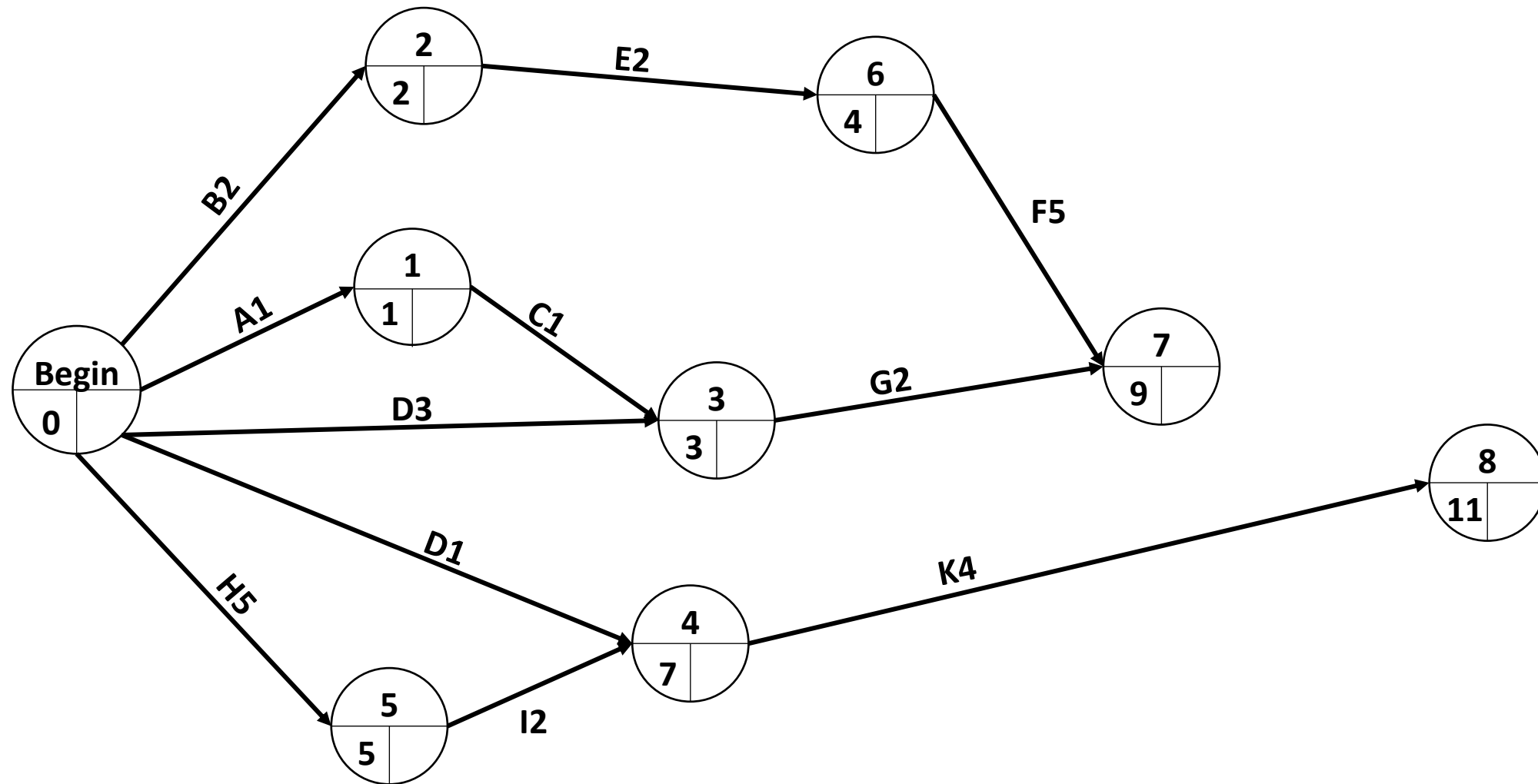
Example:

Tasks	A	B	C	D	E	F	G	H	I	I	K	L	M
Duration	1	2	1	3	2	5	2	5	2	1	4	5	4
Previous works	-	-	A	-	B	E	CD	-	H	-	I,J	F,G	K,L



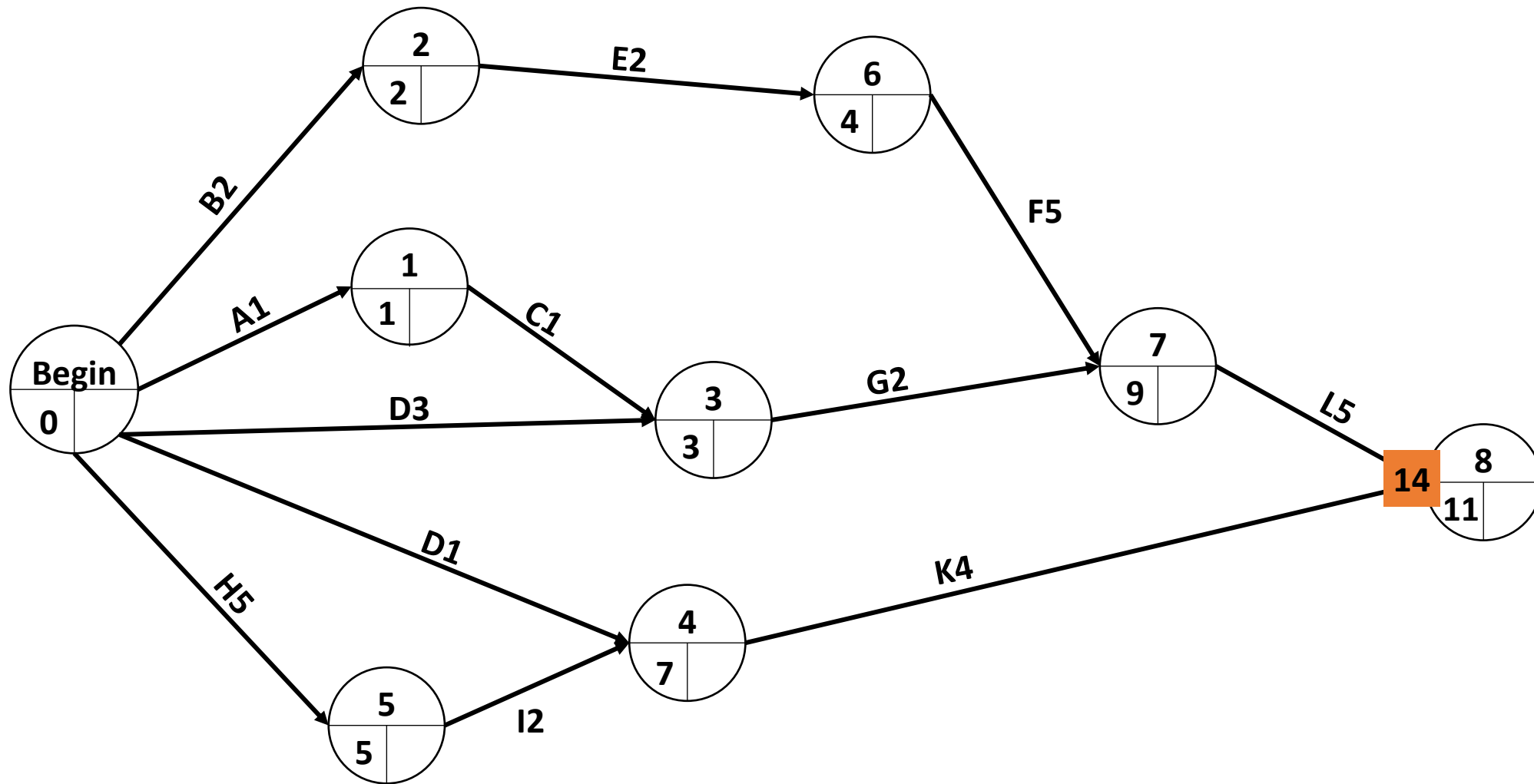
Example:

Tasks	A	B	C	D	E	F	G	H	I	I	K	L	M
Duration	1	2	1	3	2	5	2	5	2	1	4	5	4
Previous works	-	-	A	-	B	E	CD	-	H	-	I,J	F,G	K,L



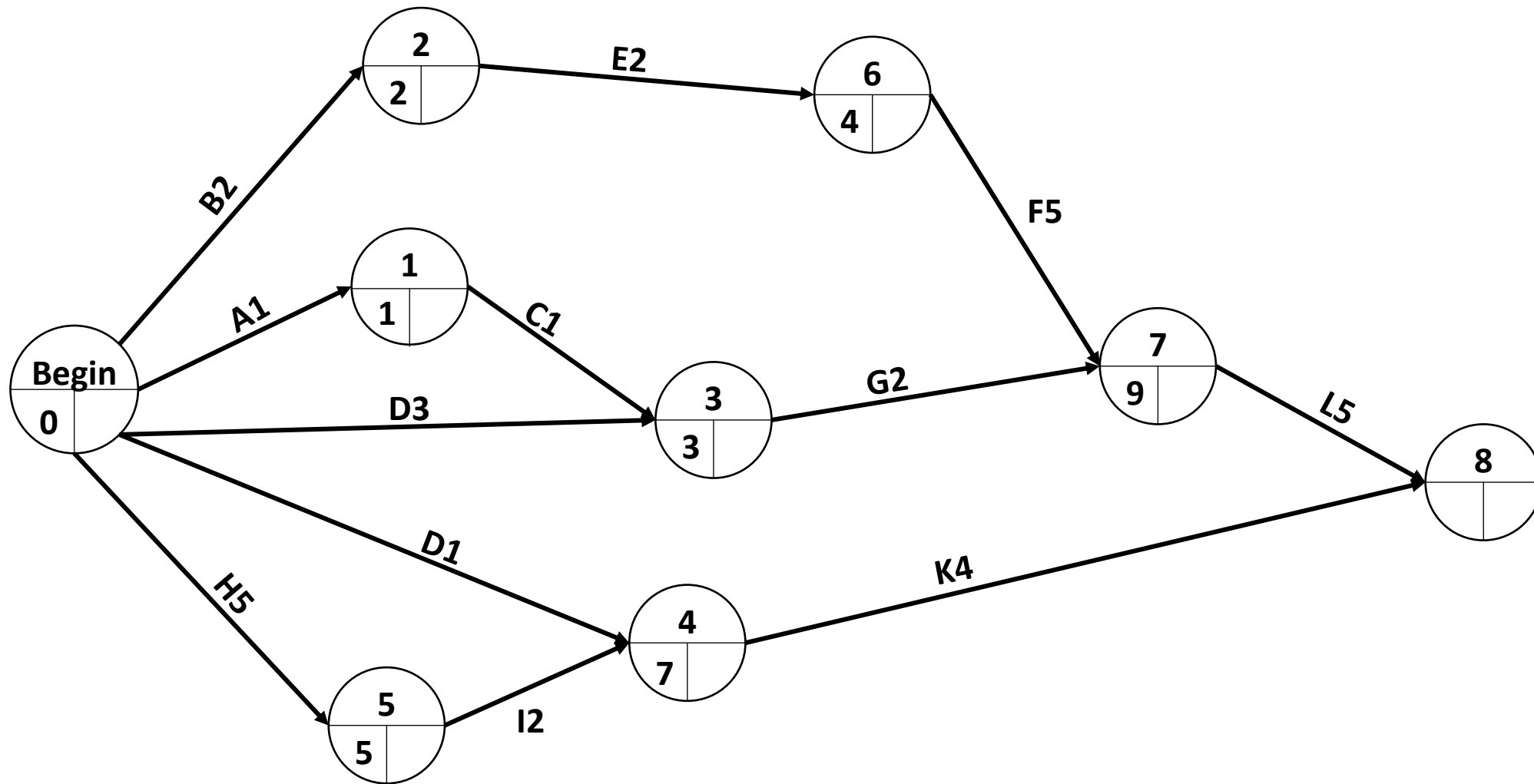
Example:

Tasks	A	B	C	D	E	F	G	H	I	I	K	L	M
Duration	1	2	1	3	2	5	2	5	2	1	4	5	4
Previous works	-	-	A	-	B	E	CD	-	H	-	I,J	F,G	K,L



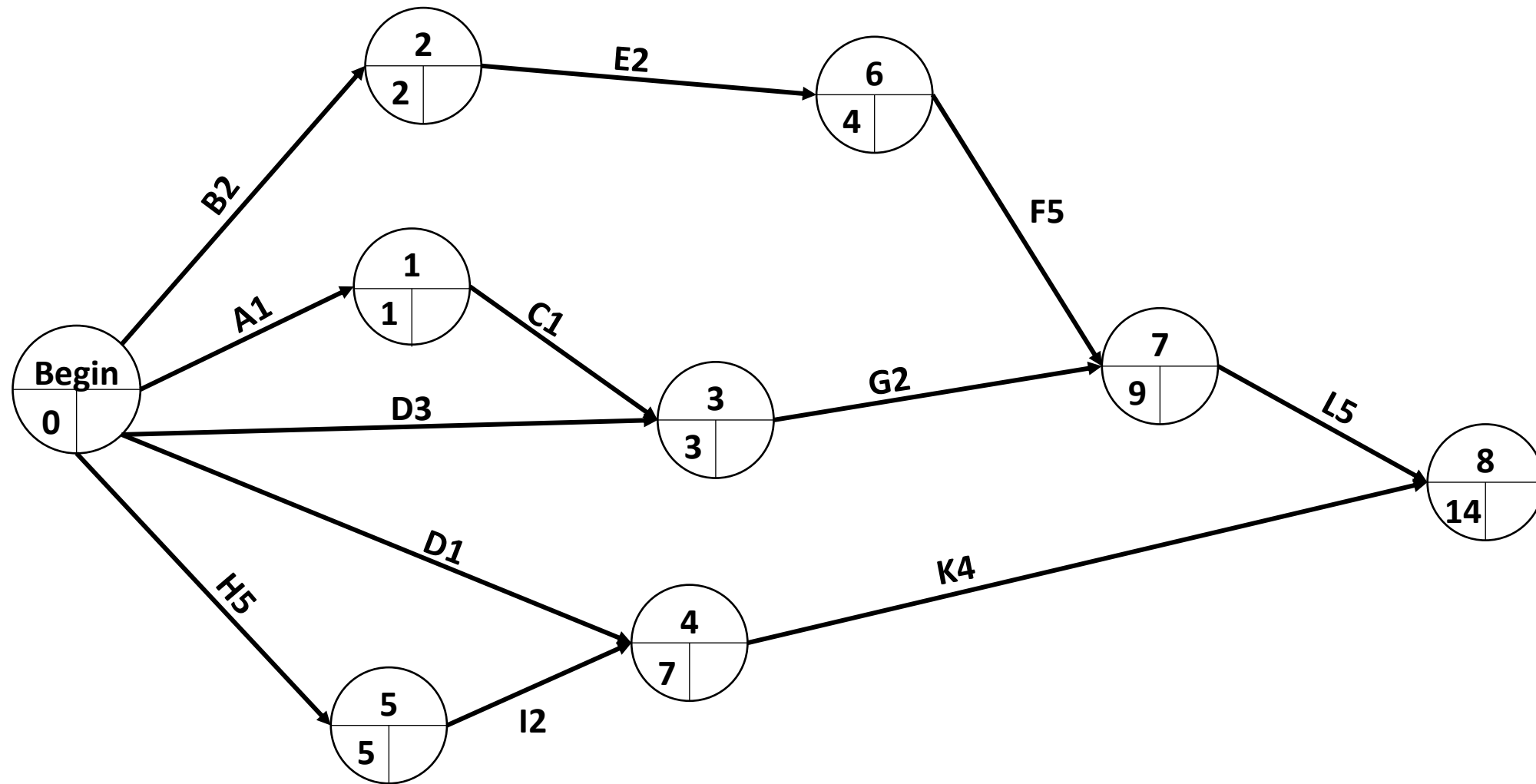
Example:

Tasks	A	B	C	D	E	F	G	H	I	I	K	L	M
Duration	1	2	1	3	2	5	2	5	2	1	4	5	4
Previous works	-	-	A	-	B	E	CD	-	H	-	I,J	F,G	K,L



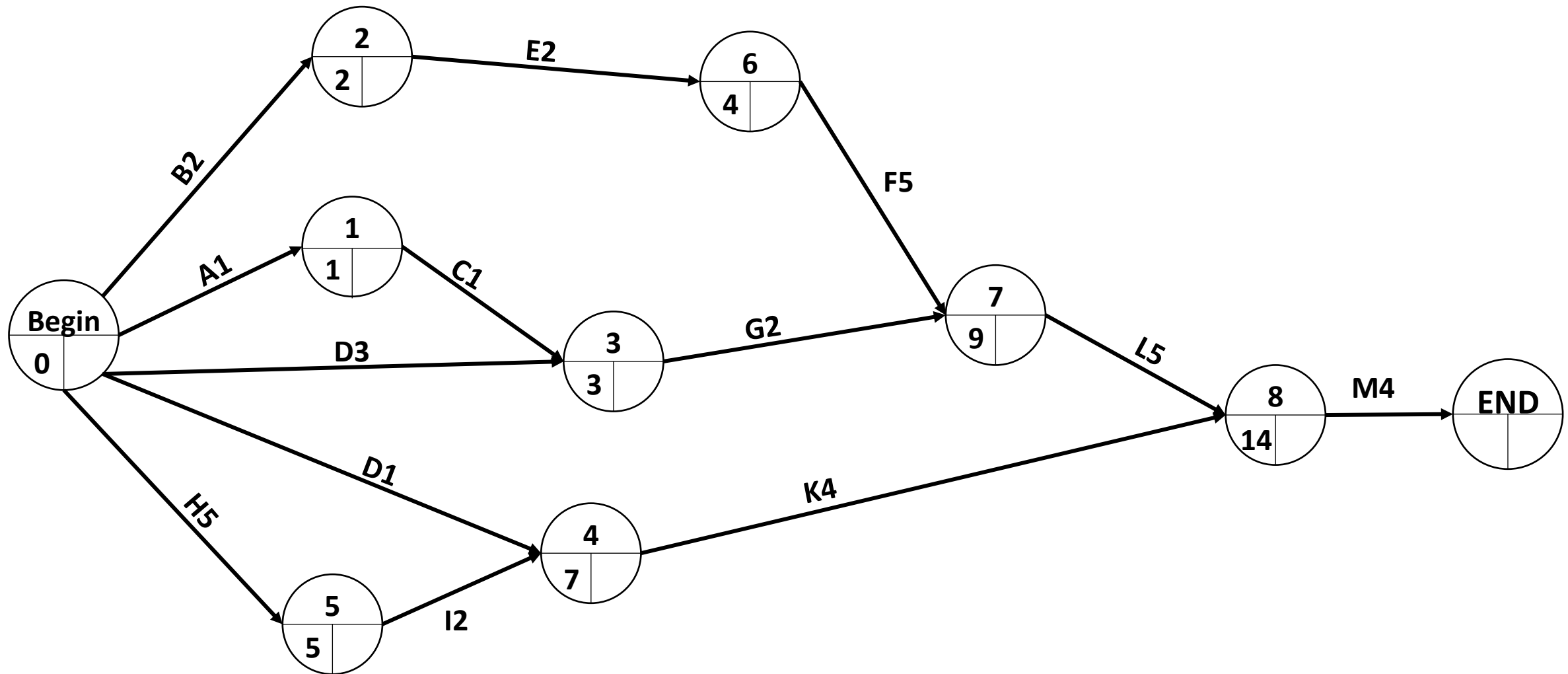
Example:

Tasks	A	B	C	D	E	F	G	H	I	I	K	L	M
Duration	1	2	1	3	2	5	2	5	2	1	4	5	4
Previous works	-	-	A	-	B	E	CD	-	H	-	I,J	F,G	K,L



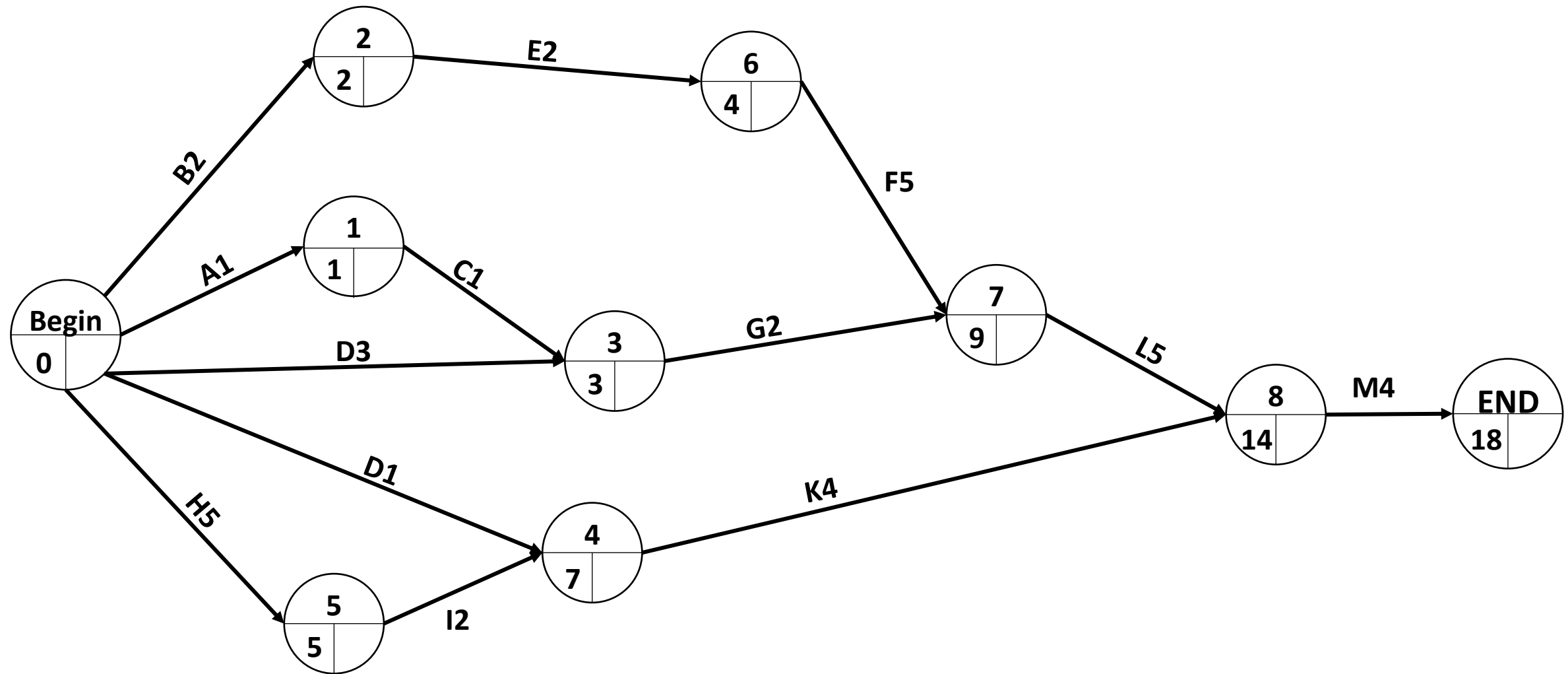
Example:

Tasks	A	B	C	D	E	F	G	H	I	I	K	L	M
Duration	1	2	1	3	2	5	2	5	2	1	4	5	4
Previous works	-	-	A	-	B	E	CD	-	H	-	I,J	F,G	K,L



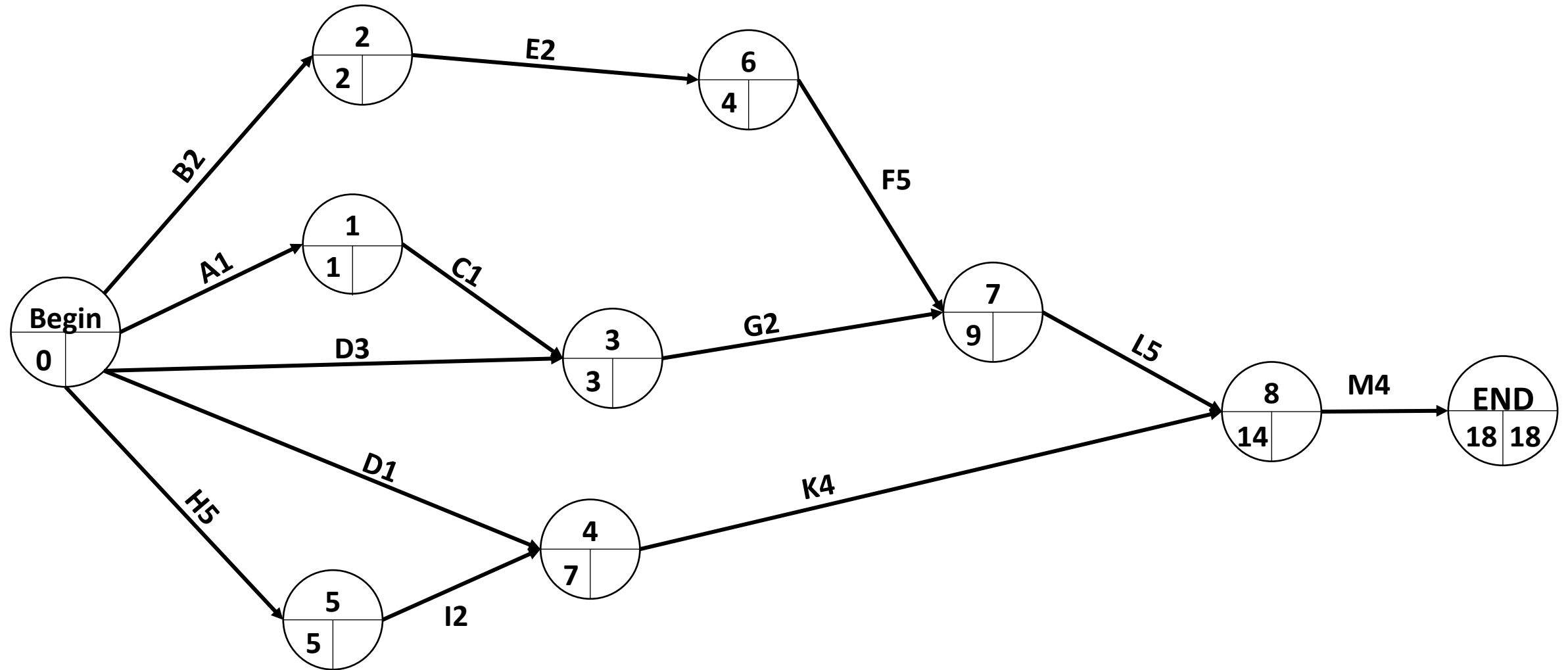
Example:

Tasks	A	B	C	D	E	F	G	H	I	I	K	L	M
Duration	1	2	1	3	2	5	2	5	2	1	4	5	4
Previous works	-	-	A	-	B	E	CD	-	H	-	I,J	F,G	K,L



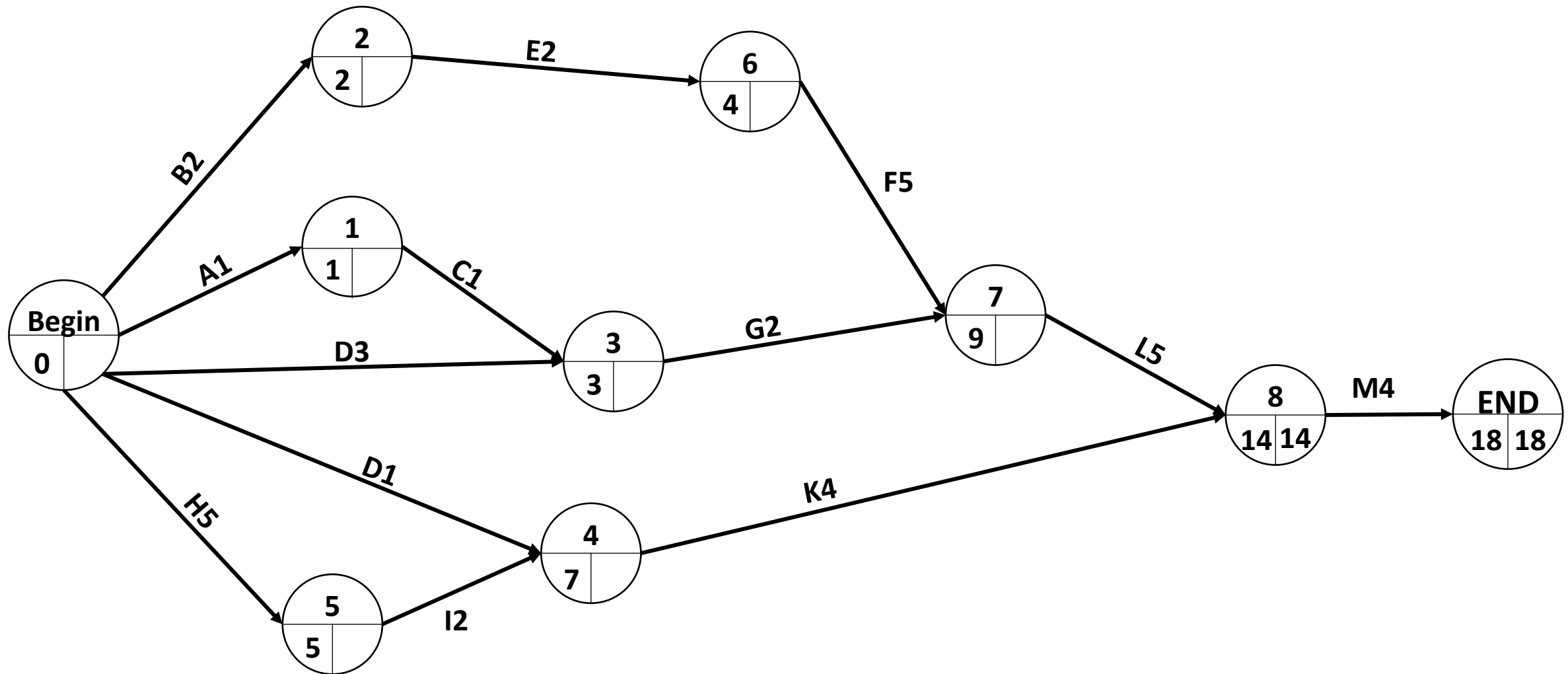
Example:

Tasks	A	B	C	D	E	F	G	H	I	I	K	L	M
Duration	1	2	1	3	2	5	2	5	2	1	4	5	4
Previous works	-	-	A	-	B	E	CD	-	H	-	I,J	F,G	K,L



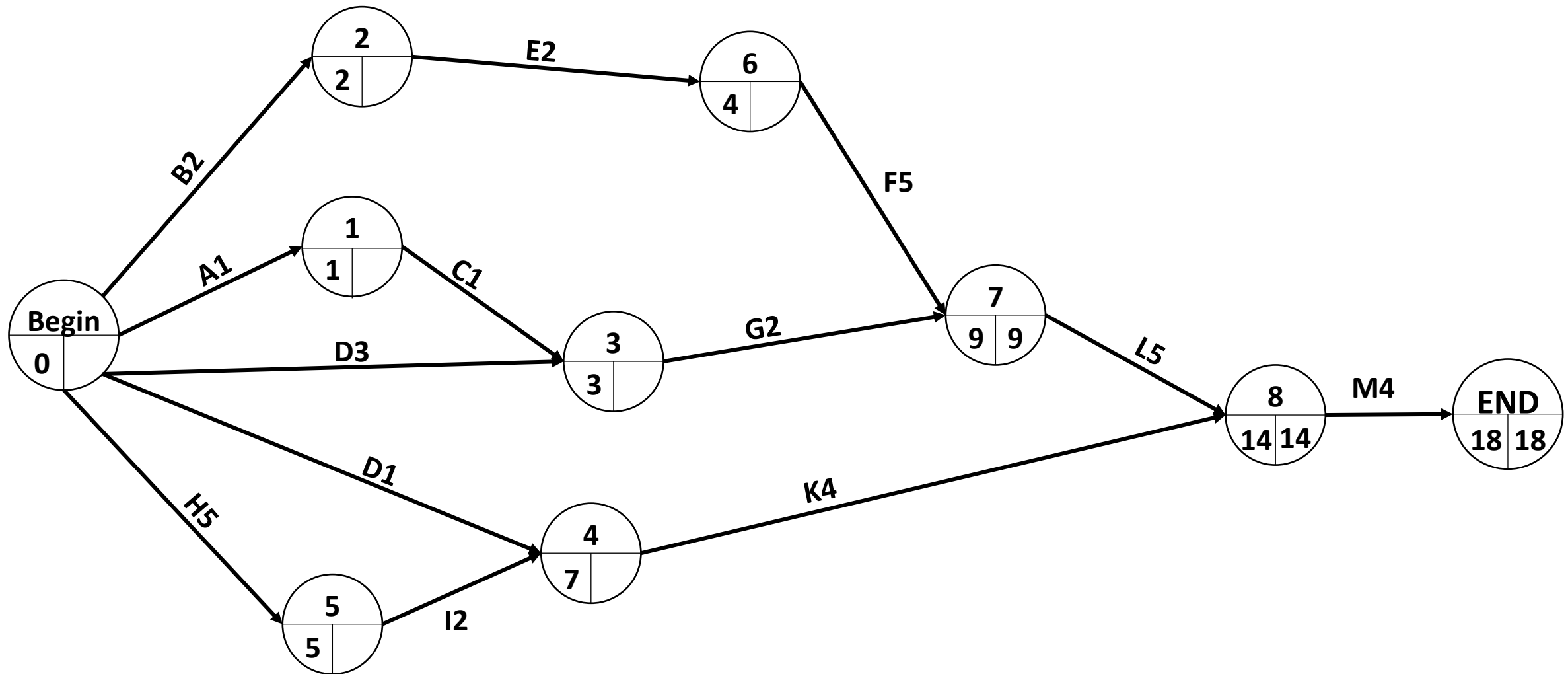
Example:

Tasks	A	B	C	D	E	F	G	H	I	I	K	L	M
Duration	1	2	1	3	2	5	2	5	2	1	4	5	4
Previous works	-	-	A	-	B	E	CD	-	H	-	I,J	F,G	K,L



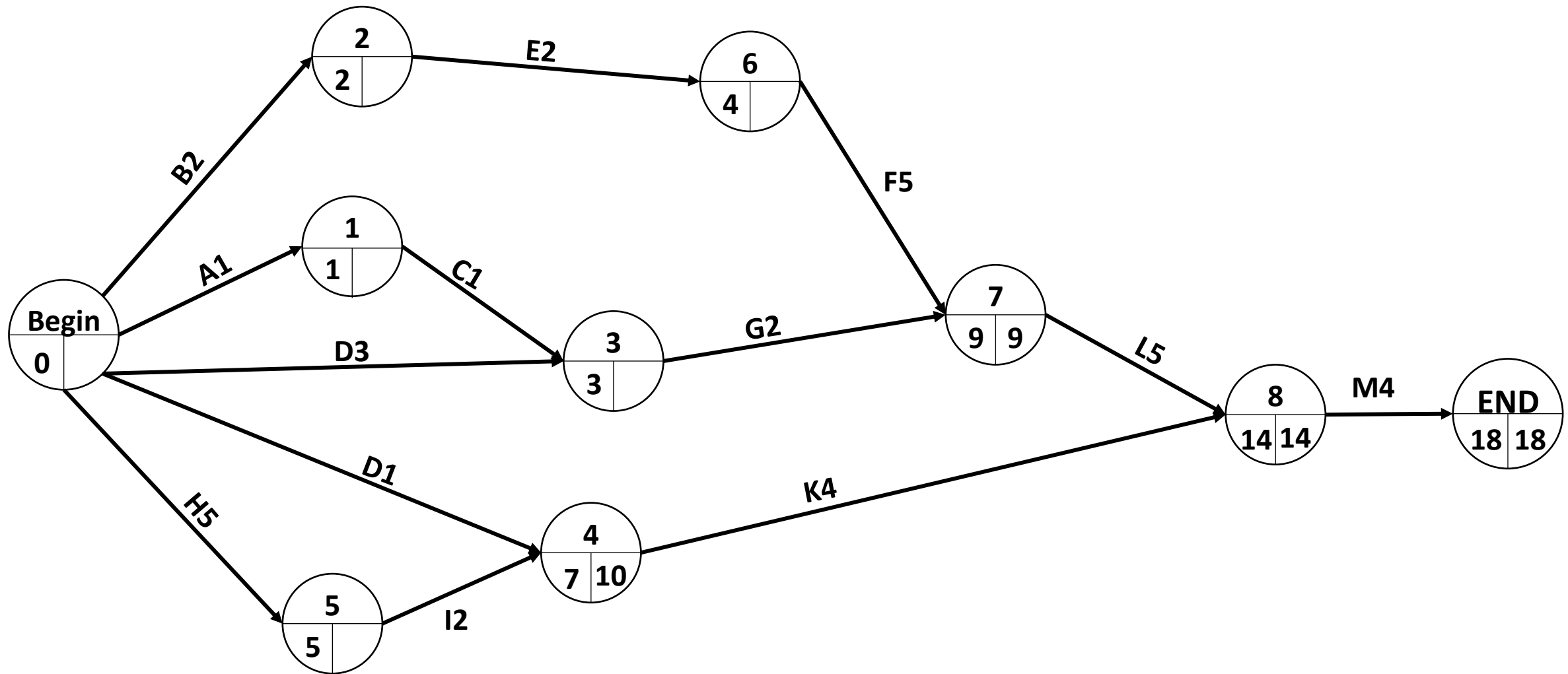
Example:

Tasks	A	B	C	D	E	F	G	H	I	I	K	L	M
Duration	1	2	1	3	2	5	2	5	2	1	4	5	4
Previous works	-	-	A	-	B	E	CD	-	H	-	I,J	F,G	K,L



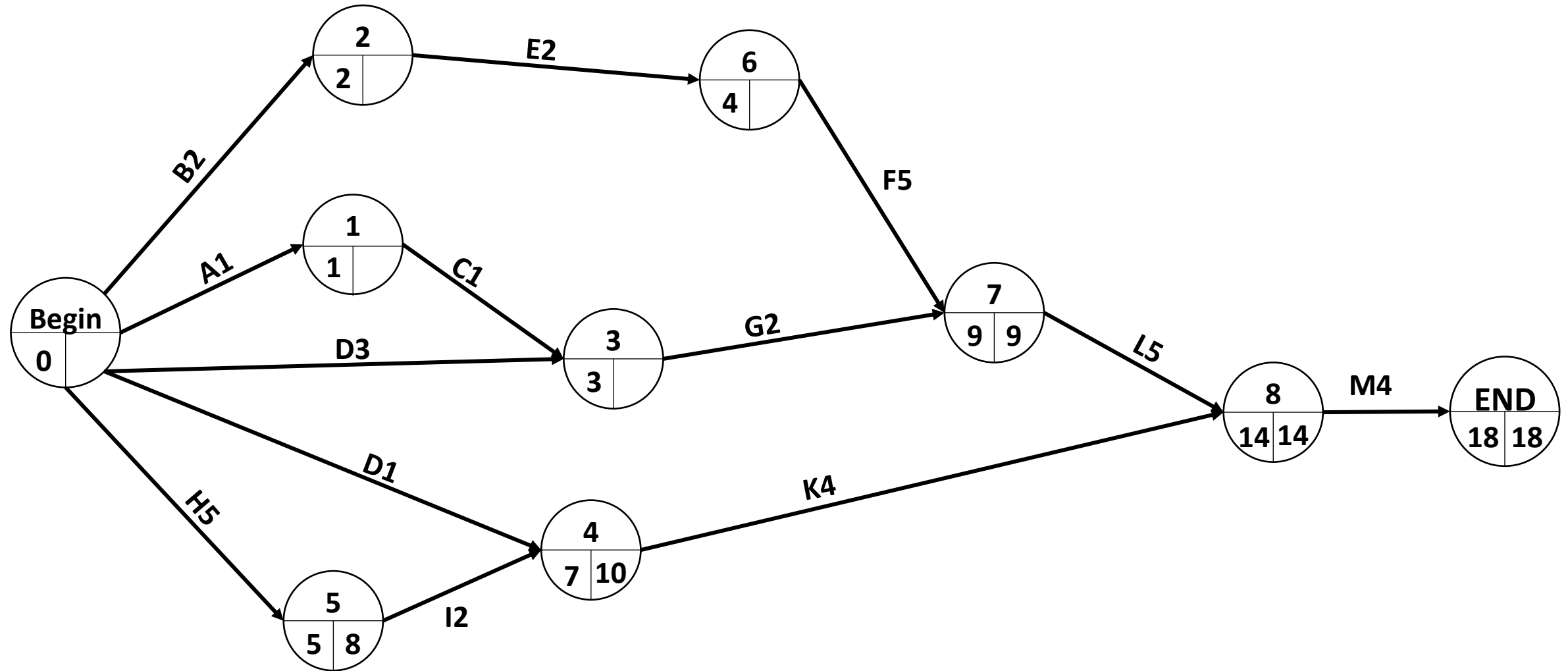
Example:

Tasks	A	B	C	D	E	F	G	H	I	I	K	L	M
Duration	1	2	1	3	2	5	2	5	2	1	4	5	4
Previous works	-	-	A	-	B	E	CD	-	H	-	I,J	F,G	K,L



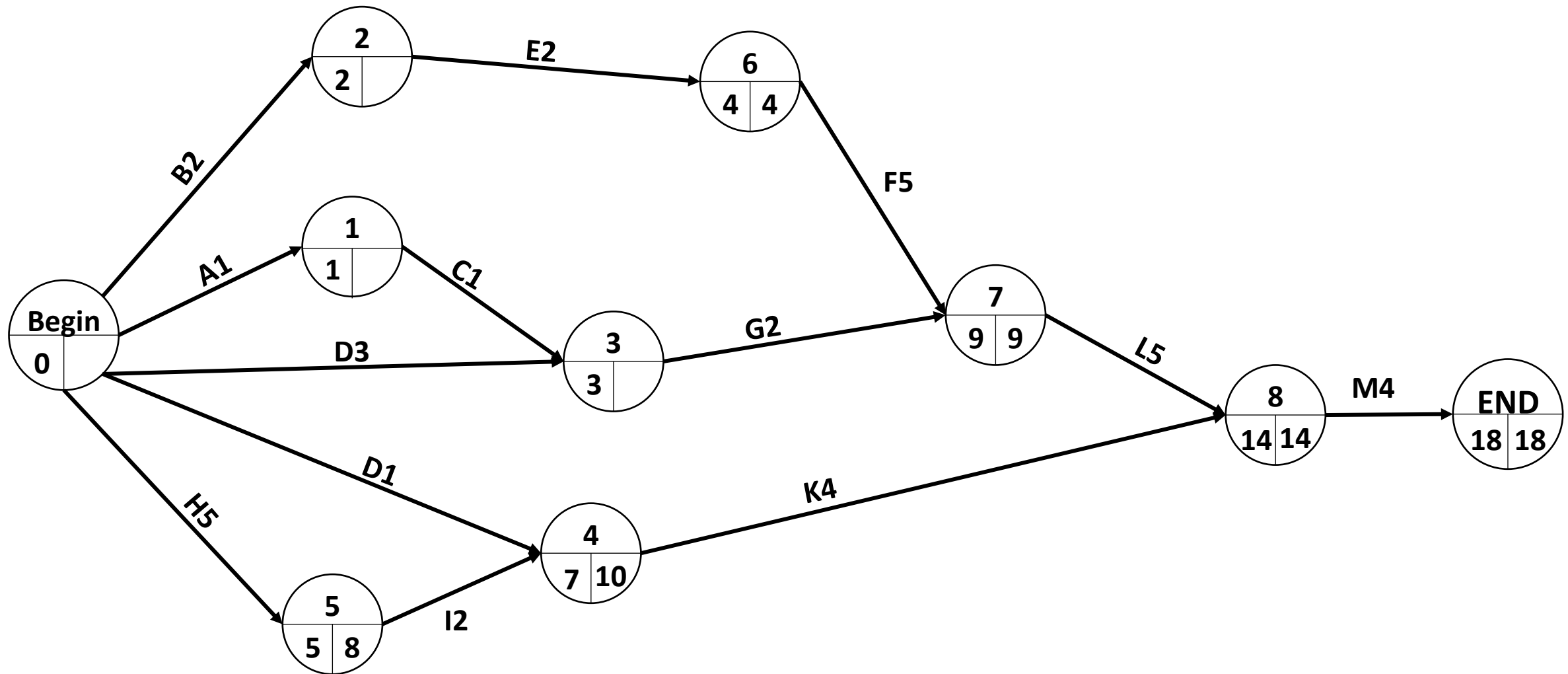
Example:

Tasks	A	B	C	D	E	F	G	H	I	I	K	L	M
Duration	1	2	1	3	2	5	2	5	2	1	4	5	4
Previous works	-	-	A	-	B	E	CD	-	H	-	I,J	F,G	K,L



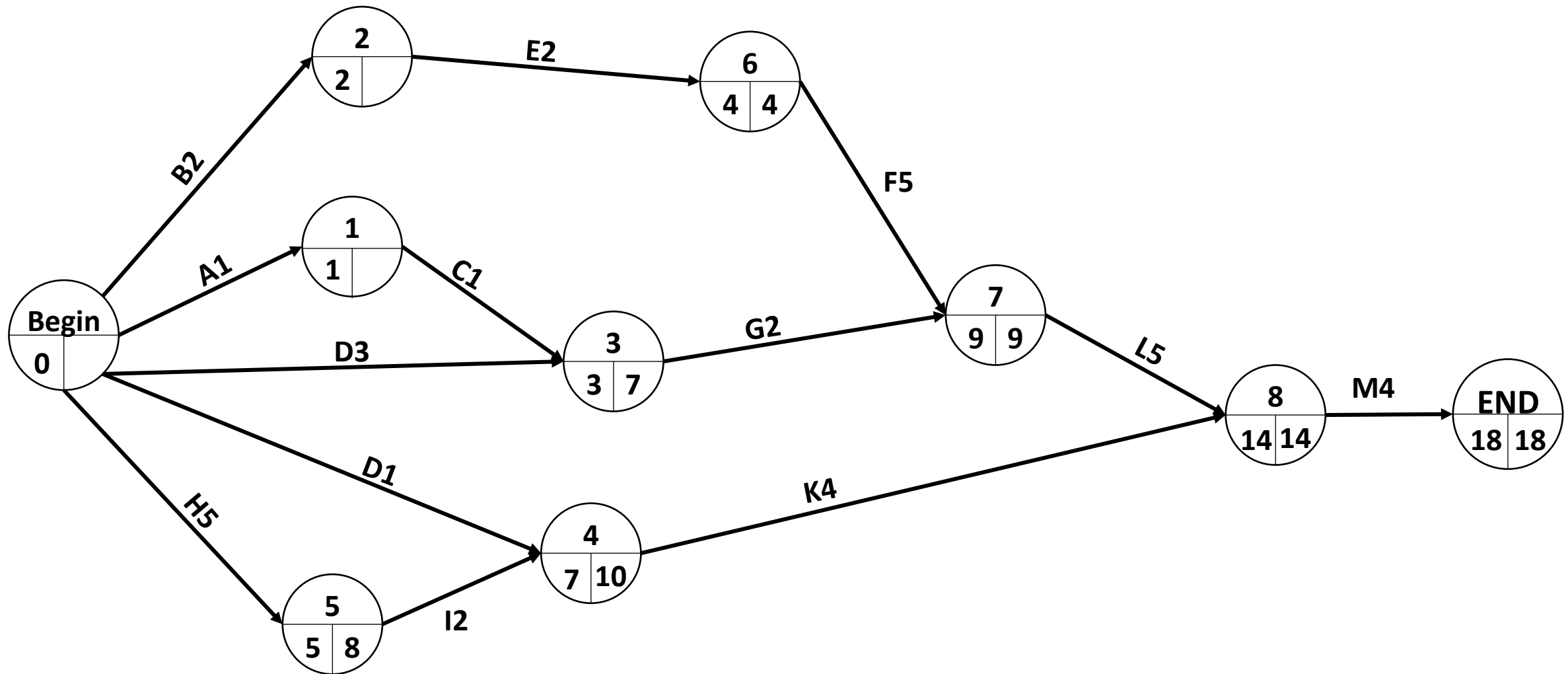
Example:

Tasks	A	B	C	D	E	F	G	H	I	I	K	L	M
Duration	1	2	1	3	2	5	2	5	2	1	4	5	4
Previous works	-	-	A	-	B	E	CD	-	H	-	I,J	F,G	K,L



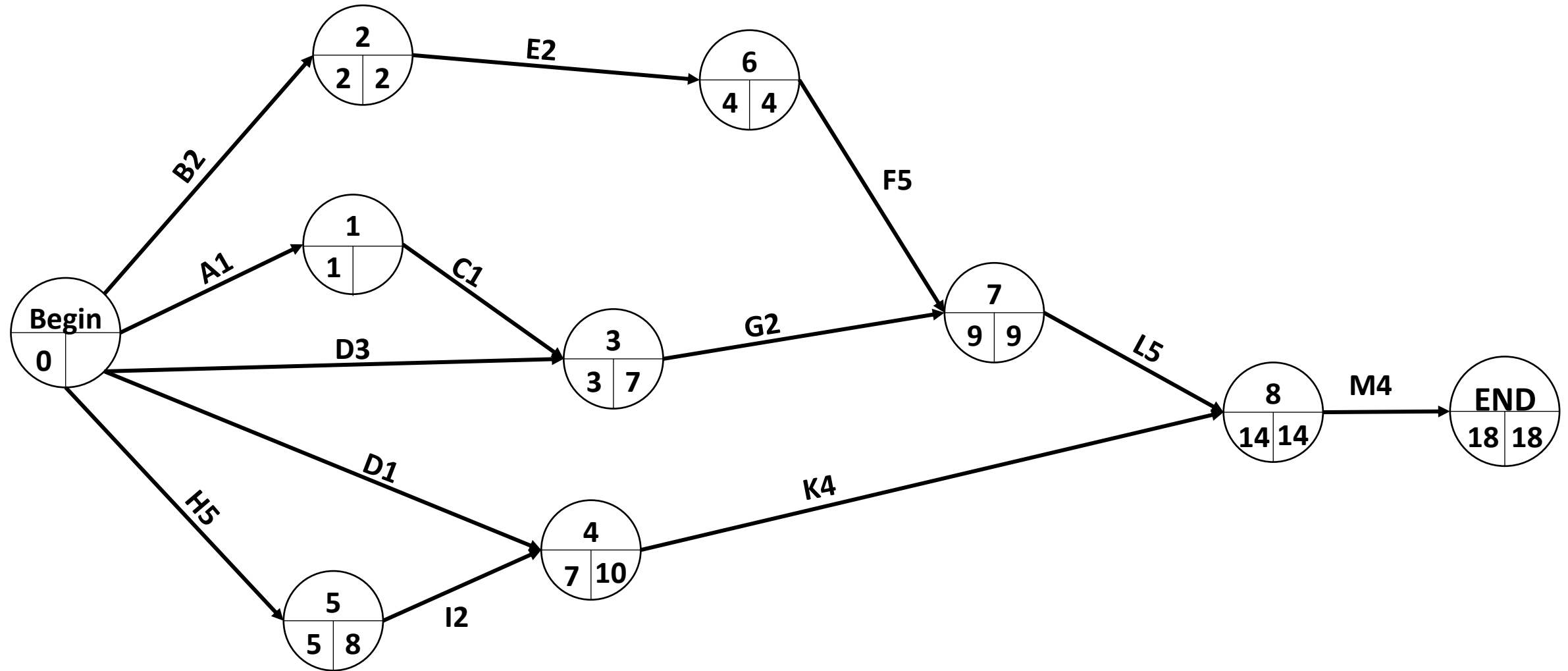
Example:

Tasks	A	B	C	D	E	F	G	H	I	I	K	L	M
Duration	1	2	1	3	2	5	2	5	2	1	4	5	4
Previous works	-	-	A	-	B	E	CD	-	H	-	I,J	F,G	K,L



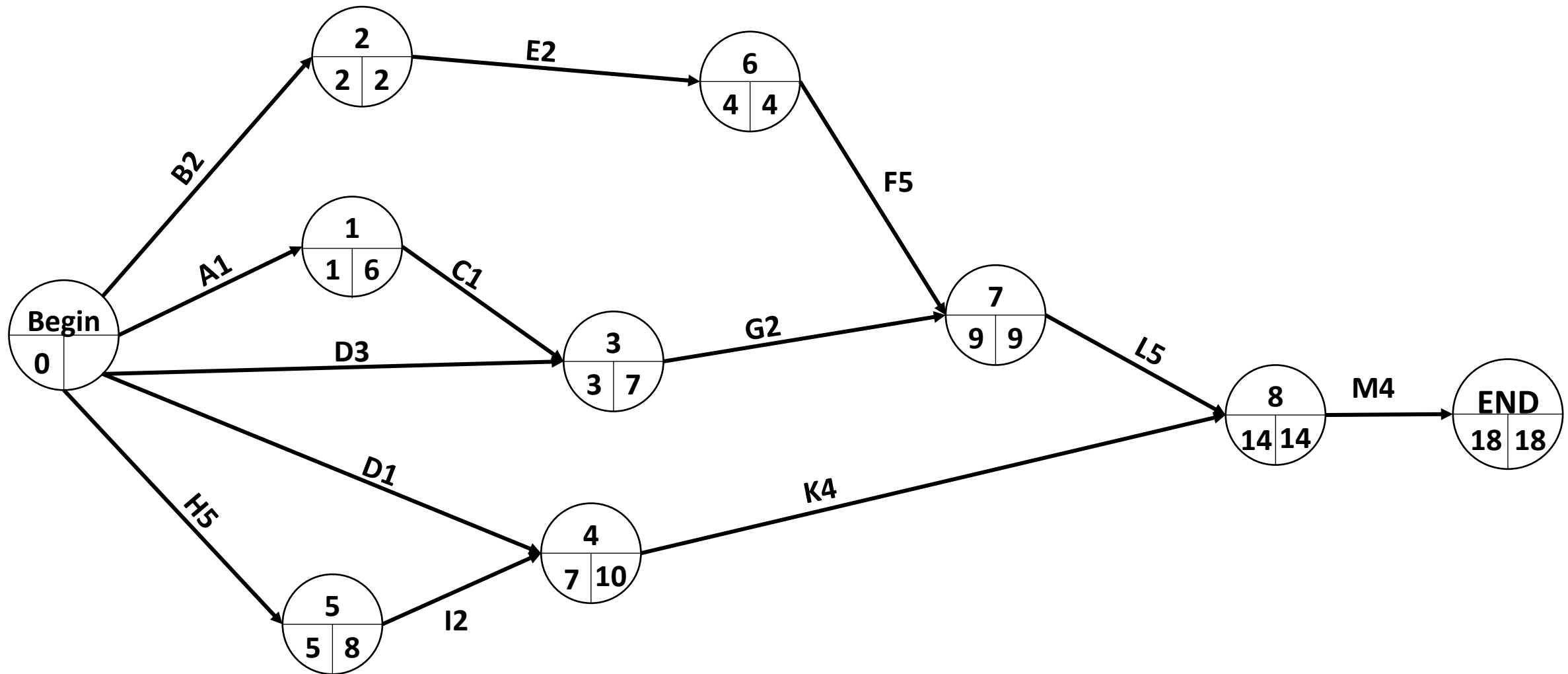
Example:

Tasks	A	B	C	D	E	F	G	H	I	I	K	L	M
Duration	1	2	1	3	2	5	2	5	2	1	4	5	4
Previous works	-	-	A	-	B	E	CD	-	H	-	I,J	F,G	K,L



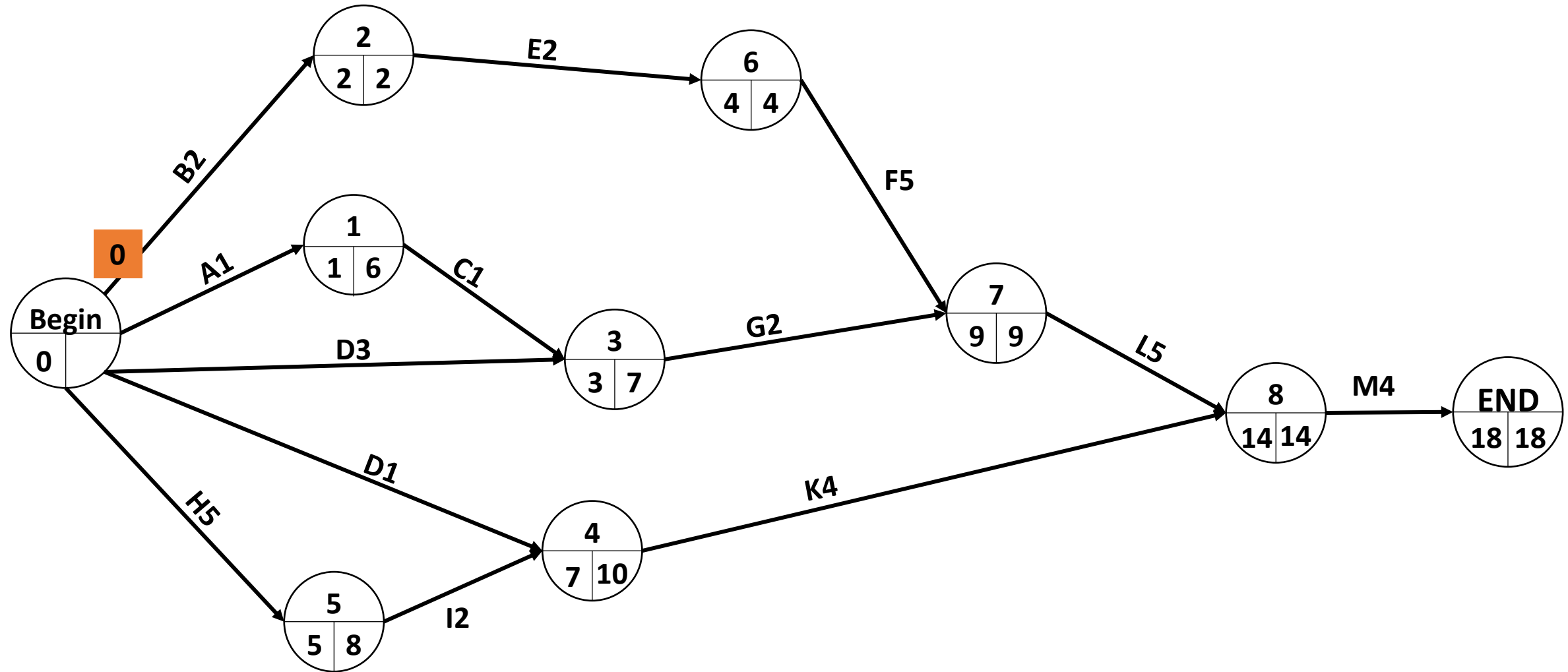
Example:

Tasks	A	B	C	D	E	F	G	H	I	I	K	L	M
Duration	1	2	1	3	2	5	2	5	2	1	4	5	4
Previous works	-	-	A	-	B	E	CD	-	H	-	I,J	F,G	K,L



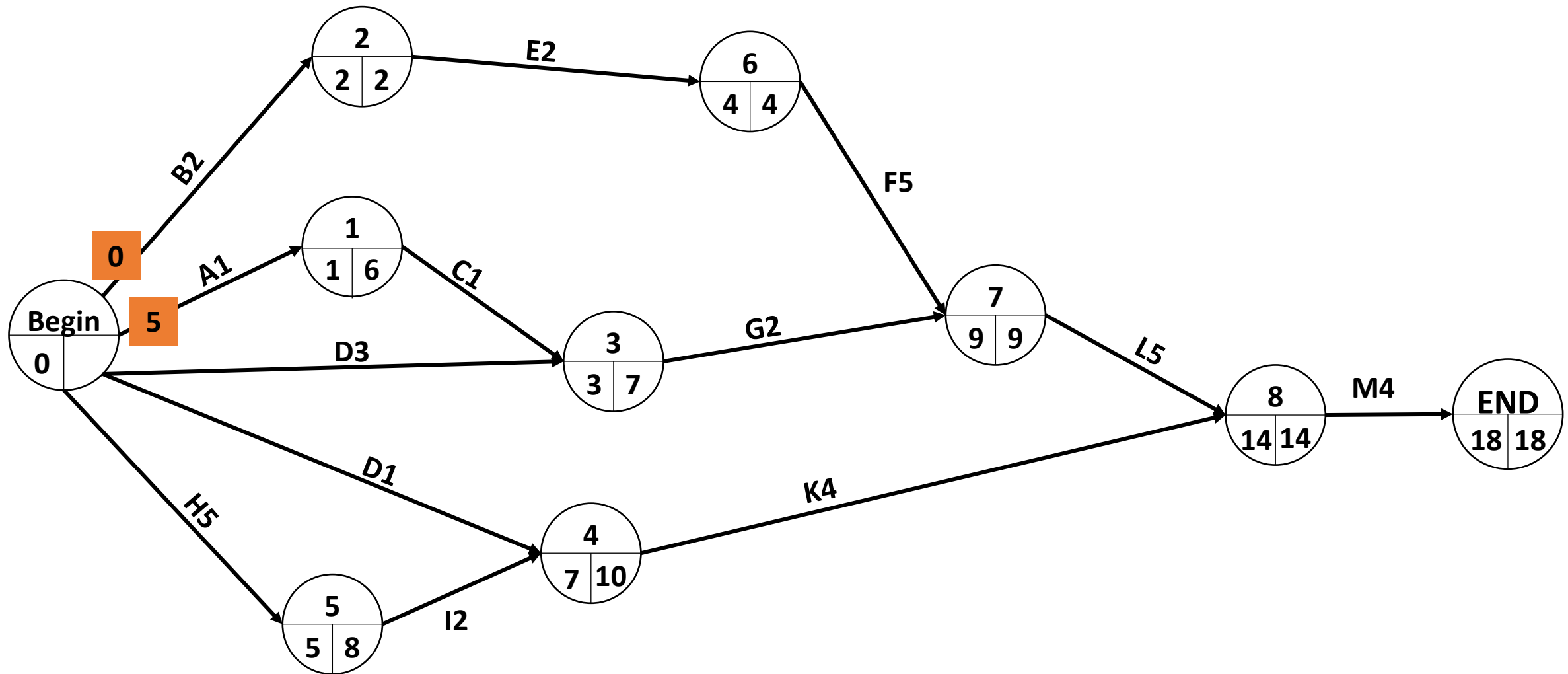
Example:

Tasks	A	B	C	D	E	F	G	H	I	I	K	L	M
Duration	1	2	1	3	2	5	2	5	2	1	4	5	4
Previous works	-	-	A	-	B	E	CD	-	H	-	I,J	F,G	K,L



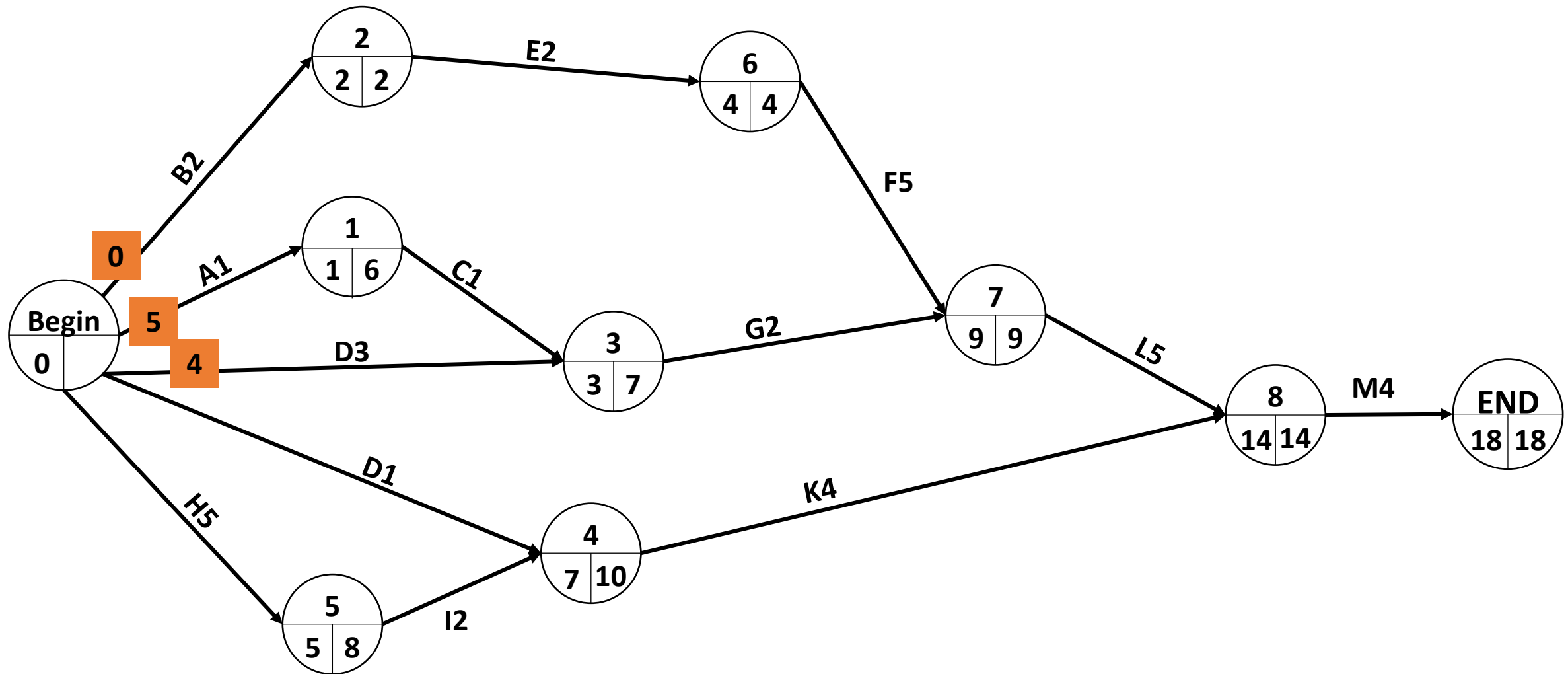
Example:

Tasks	A	B	C	D	E	F	G	H	I	I	K	L	M
Duration	1	2	1	3	2	5	2	5	2	1	4	5	4
Previous works	-	-	A	-	B	E	CD	-	H	-	I,J	F,G	K,L



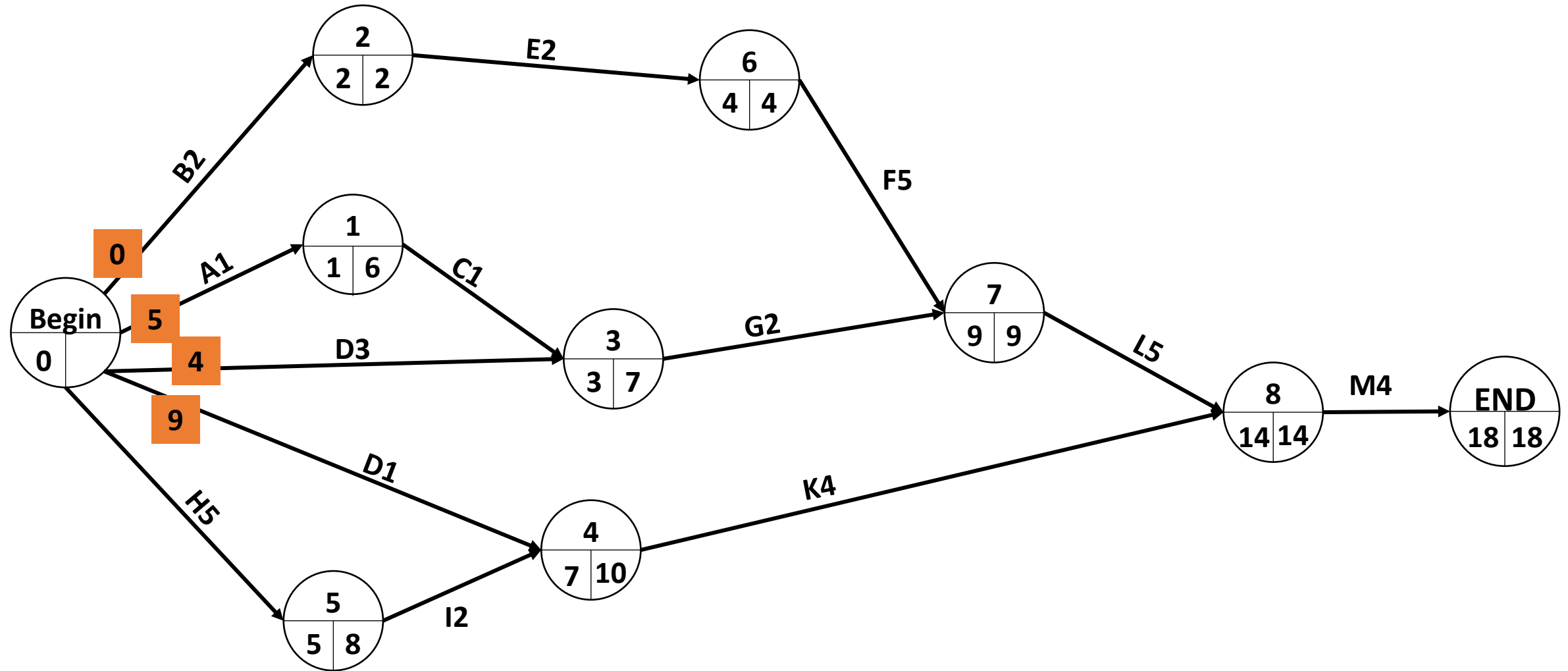
Example:

Tasks	A	B	C	D	E	F	G	H	I	I	K	L	M
Duration	1	2	1	3	2	5	2	5	2	1	4	5	4
Previous works	-	-	A	-	B	E	CD	-	H	-	I,J	F,G	K,L



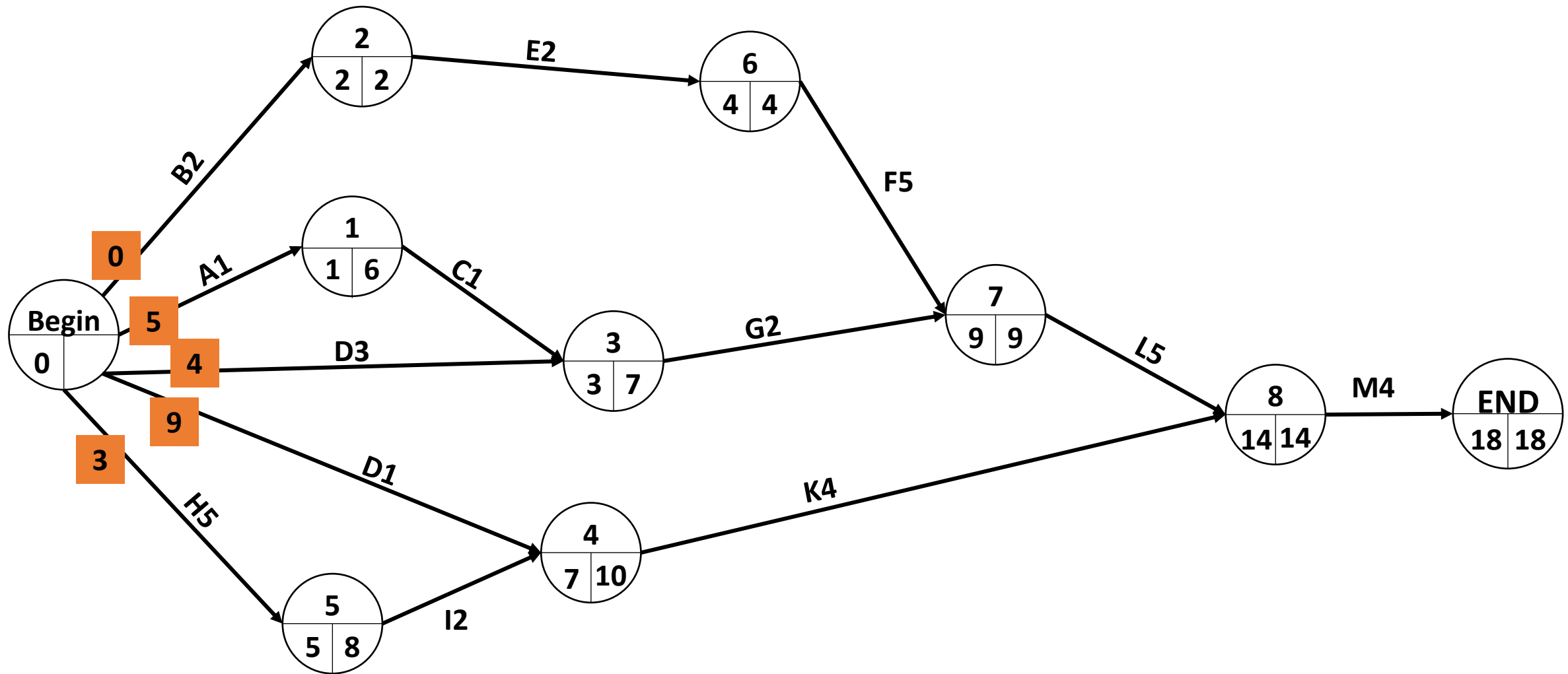
Example:

Tasks	A	B	C	D	E	F	G	H	I	I	K	L	M
Duration	1	2	1	3	2	5	2	5	2	1	4	5	4
Previous works	-	-	A	-	B	E	CD	-	H	-	I,J	F,G	K,L



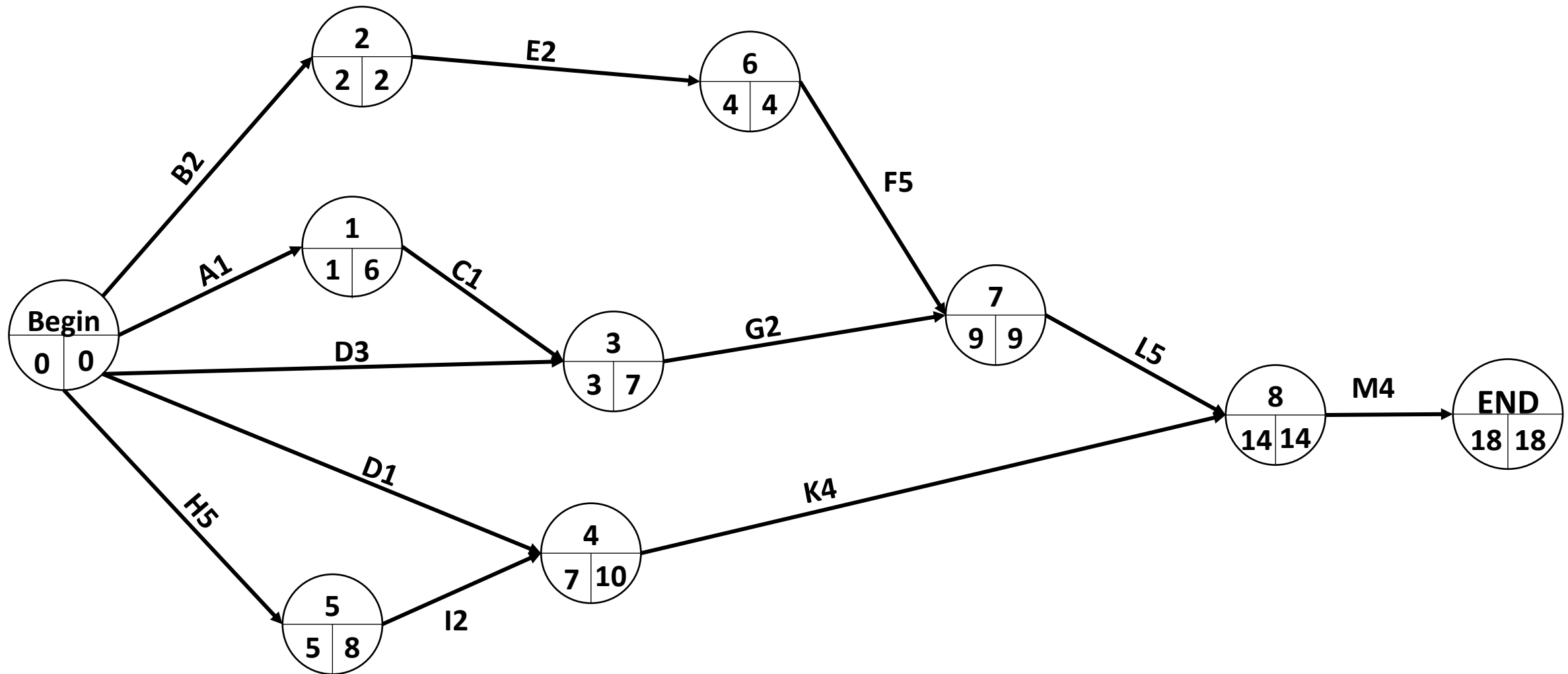
Example:

Tasks	A	B	C	D	E	F	G	H	I	I	K	L	M
Duration	1	2	1	3	2	5	2	5	2	1	4	5	4
Previous works	-	-	A	-	B	E	CD	-	H	-	I,J	F,G	K,L



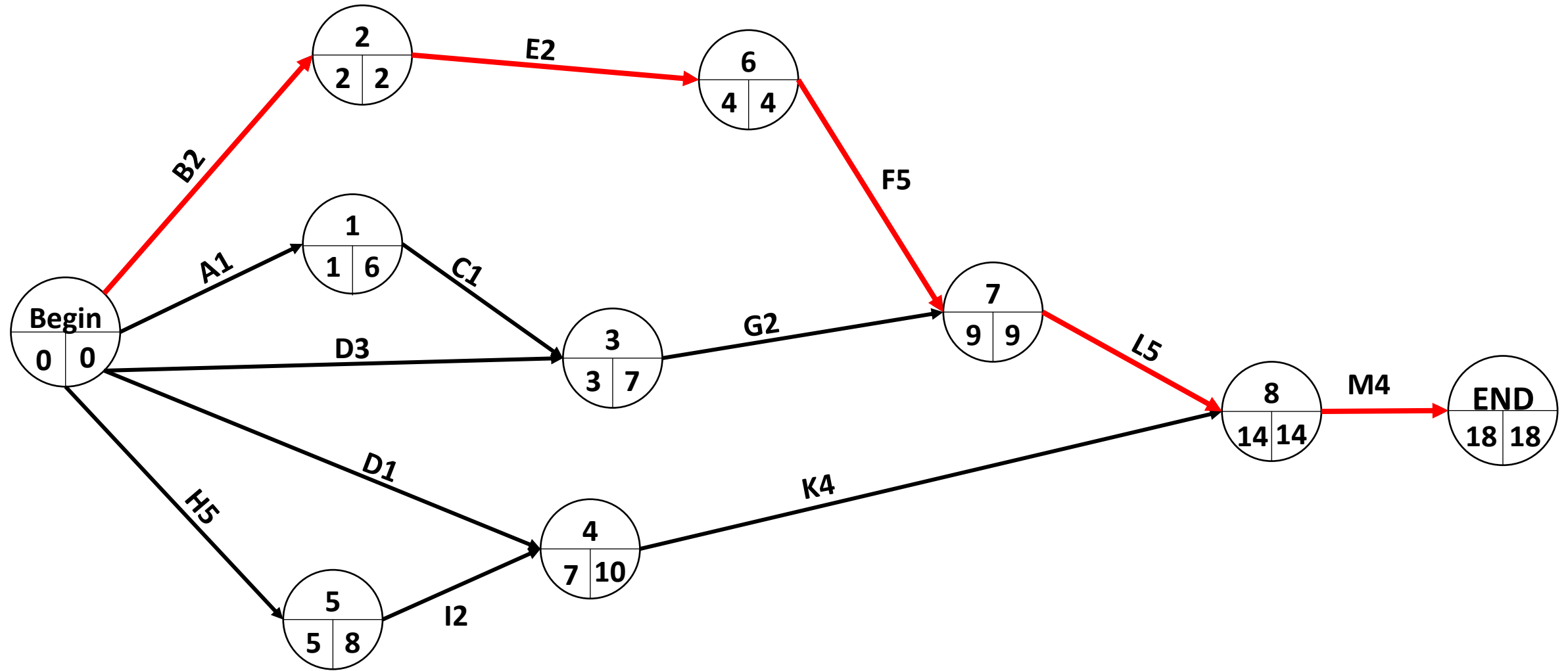
Example:

Tasks	A	B	C	D	E	F	G	H	I	I	K	L	M
Duration	1	2	1	3	2	5	2	5	2	1	4	5	4
Previous works	-	-	A	-	B	E	CD	-	H	-	I,J	F,G	K,L



Example:

Tasks	A	B	C	D	E	F	G	H	I	I	K	L	M
Duration	1	2	1	3	2	5	2	5	2	1	4	5	4
Previous works	-	-	A	-	B	E	CD	-	H	-	I,J	F,G	K,L



Give GANTT and PERT chart

Stain	Duration	Background(s)
A	2	-
B	8	-
C	5	A
D	2	B
E	6	B
F	5	E
G	3	A,D

