Exercises Series N°: 3

Exercise 1: Consider the simple graph whose vertices are the natural integers between 1 and 20, and such that two vertices i and j are connected if and only if $i + j <= 21$.

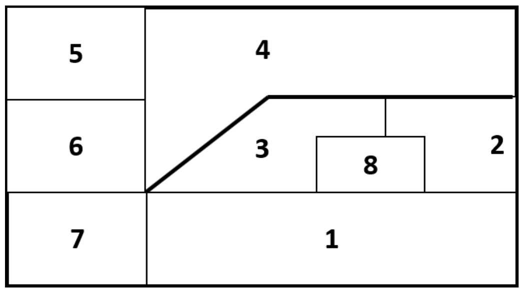
1. Prove that this graph is connected.
2. Find its diameter (you will remember the definition of diameter).

Exercise 2: In a game, two players have two piles of three matches each. Each turn, a player can remove one or two matches. The player who removes the last match loses the game.

1. Model this game using a graph.
2. Determine the winning strategy for the first player to guarantee victory.

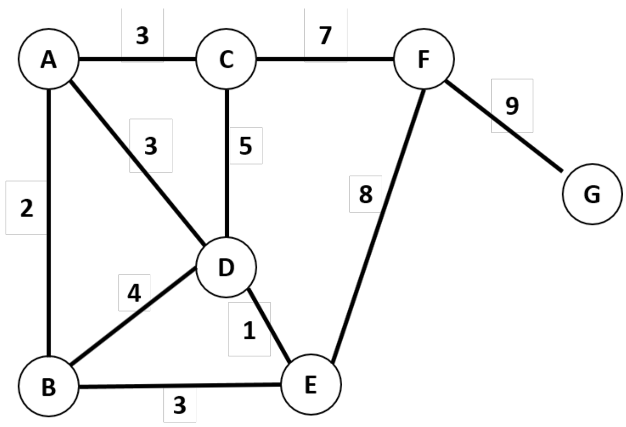
Exercise 3: In a group of six people, show that there must be either three people who all know each other or three people who all do not know each other (assuming that if A knows B, then B also knows A).
Then, show that this statement does not necessarily hold for a group of five people.

Exercise 4: We are given a map of eight countries and their borders. Two countries are considered adjacent if they share a border, except when the shared border is a finite number of points (e.g., touching only at corners).



1. Represent the adjacency graph.
2. Is the graph complete? Is it connected? What is the degree of each vertex? How many edges are in the graph?
3. What is the distance between vertices 1 and 5? What is the diameter of the graph?
4. Is it possible to start in one country and return to the same country after crossing each border exactly once (Eulerian circuit)? Is it possible to start in one country, cross each border exactly once, and end up in a different country (Eulerian path)?
5. What is the maximum number of countries with no common border?
6. Color the eight countries with the minimum number of colors such that two adjacent countries have different colors.

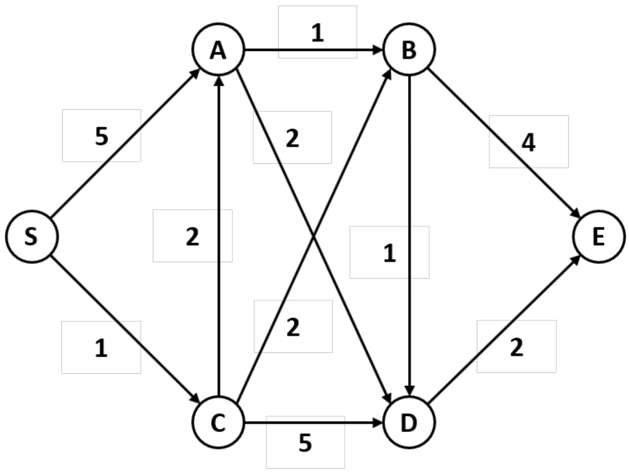
Exercise 5: Consider the following graph:

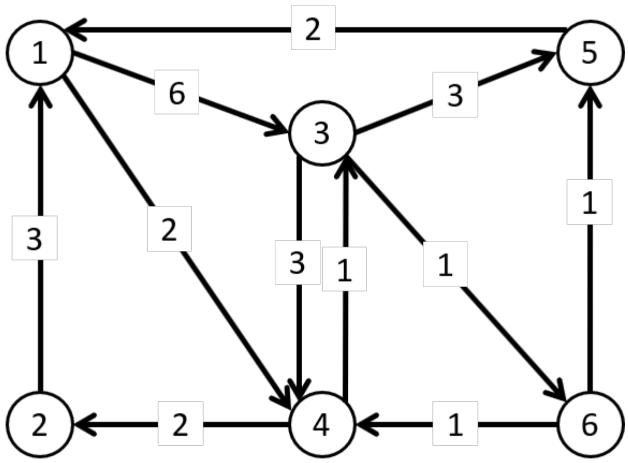


1. Apply Prim's algorithm to find the minimum spanning tree.
2. Kruskal's algorithm to find the minimum spanning tree.

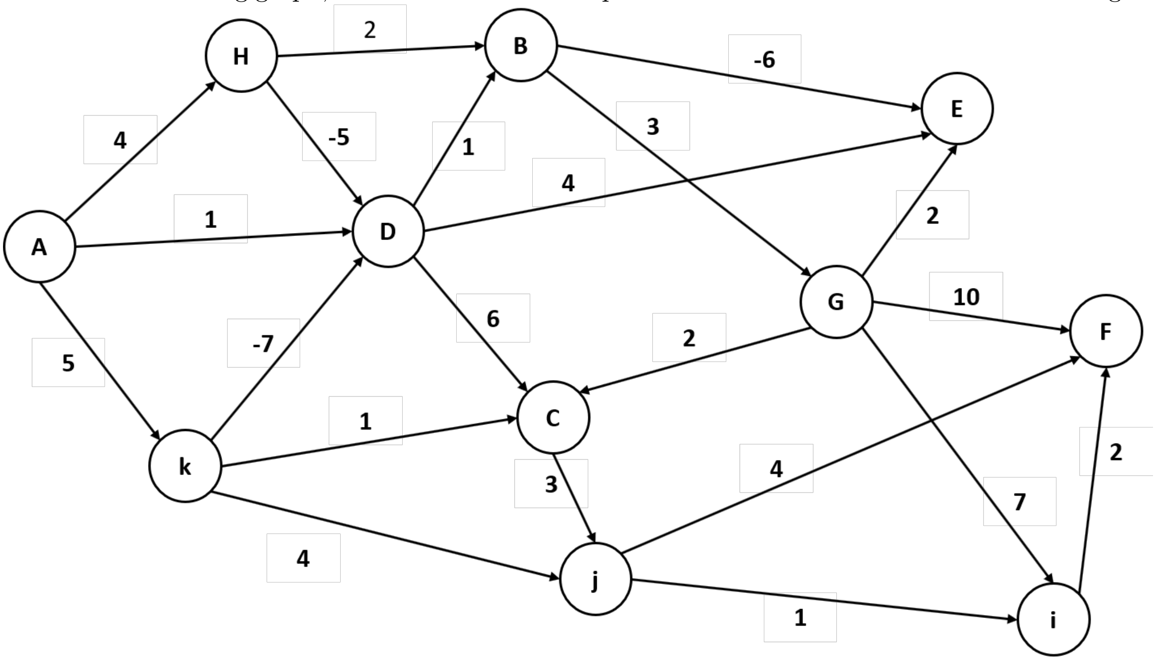
Exercise 6:

1. Explain why edges with negative weights can cause problems in finding a shortest path in a graph.
2. Apply Dijkstra's algorithm to the following graphs to find the shortest paths from s to the other vertices.





Exercise 7: Given the following graph, determine the shortest paths from vertex a to all other vertices using the appropriate



algorithm.

Exercise 8: An airline flies to different cities. The following table gives the flight times between these cities.

	A	B	C	D	E
A		1h30	2h		2h15
B	1h40				3h
C	2h20			2h22	
D			3h20		1h05
E	2h25	3h10	1h10		

1. How to determine the fastest route between two cities?
2. Modify the proposed method to take into account the duration of stopovers in different cities.